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Forbidden pairs for k-connected Hamiltonian graphs

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ABSTRACT

For an integer k with $k \geq 2$ and a pair of connected graphs F_1 and F_2 of order at least three, we say that $\{F_1, F_2\}$ is a k-forbidden pair if every k-connected $\{F_1, F_2\}$-free graph, except possibly for a finite number of exceptions, is Hamiltonian. If no exception arises, $\{F_1, F_2\}$ is said to be a strong k-forbidden pair. The 2-forbidden pairs and the strong 2-forbidden pairs are determined by Faudree and Gould (1997) [11] and Bedrossian (1991) [1], respectively. All of them contain $K_{1,3}$. In this paper, we prove that $\{K_{1,k+1}, P_4\}$ is a strong k-forbidden pair, which shows that $K_{1,3}$ is not always necessary in a k-forbidden pair for $k \geq 3$. On the other hand, we prove that each k-forbidden pair contains $K_{1,l}$ for some $l \leq k+1$. We also discuss several other Hamiltonian properties of k-connected $\{K_{1,k+1}, P_4\}$-free graphs.

1. Introduction

We will generally follow West [21] for terminology and notation, and consider simple graphs only. For a connected graph F, a graph G is said to be F-free if it does not contain an induced subgraph isomorphic to F. The graph F is called a forbidden subgraph. More generally, for a set of connected graphs $\{F_1, F_2, \ldots, F_k\}$, a graph G is said to be $\{F_1, F_2, \ldots, F_k\}$-free if G is F_i-free for each i, $1 \leq i \leq k$. The graphs $F_1, \ldots, F_k$ are called forbidden subgraphs. In order to avoid triviality, we assume that all forbidden subgraphs have order at least three.

Hamiltonicity involving a set of two or three forbidden subgraphs has been studied actively for years. We refer the reader to Faudree [9] for a survey of this topic. Bedrossian [1] characterized all the pairs of forbidden subgraphs that force the existence of a Hamiltonian cycle in a 2-connected graph without any exception. Later Faudree and Gould [11] extended it and characterized all the pairs of forbidden subgraphs that force a 2-connected graph to be Hamiltonian, allowing finitely many exceptions. These two papers opened up a new vista in the research of Hamiltonicity and forbidden subgraphs. In [5,12,14], the characterization of the triples forcing Hamiltonicity in a 2-connected graph was completed under the assumption that no exception is allowed. The same problem was addressed in [12,13] under the assumption that finitely many exceptions are allowed, but a complete characterization has not been obtained yet. A similar problem for Hamiltonian-connectedness was discussed in [3,6]. While every Hamiltonian-connected graph is 3-connected, a characterization of forbidden pairs for Hamiltonian-connectedness in the class of 3-connected graphs appears to be far from completed.

In this type of problem, if we impose higher connectivity in the assumption, the class of graphs in consideration becomes smaller and hence a weaker condition may guarantee Hamiltonian properties. Therefore, if we raise the connectivity, the

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number of forbidden pairs or triples may increase. It actually happens for 4-connected graphs. In [4], Broersma et al. gave a pair of forbidden subgraphs which guarantees the existence of a Hamiltonian cycle in a 4-connected graph. Later Pfender [18] extended this result by giving another pair. Neither pair guarantees Hamiltonicity of a 2-connected graph, hence they do not belong to the set of 2-forbidden pairs determined in [11]. Note that both pairs contain $K_{1,3}$.

In this paper, we consider pairs of forbidden subgraphs that force Hamiltonicity in a graph of high connectivity. Let k be an integer with $k \geq 2$, and let F_1 and F_2 be connected graphs of order at least three. Then $\{F_1, F_2\}$ is called a *k-forbidden pair* for Hamiltonian graphs, or simply a k-forbidden pair, if there exists a positive integer N such that every k-connected $\{F_1, F_2\}$-free graph of order at least N is Hamiltonian. If we can take $N = k + 1$ in this definition, or equivalently every k-connected $\{F_1, F_2\}$-free graph is Hamiltonian without any exception, then $\{F_1, F_2\}$ is called a *strong* k-forbidden pair. The ultimate goal would be to characterize all k-forbidden pairs for every k, but considering the long-standing conjecture by Matthews and Sumner [17], we are not optimistic. We will discuss this in the concluding remarks.

With the background stated above, the purpose of this paper is to investigate k-forbidden pairs for $k \geq 3$ and observe the difference from 2-forbidden pairs. According to the characterization in [11], each 2-forbidden pair contains a claw. To show a contrast, we first remark that for $k \geq 3$, there exists a strong k-forbidden pair which does not contain a claw. We then prove that for each $k \geq 3$, every k-forbidden pair contains a star with at most $k + 1$ leaves.

As we wrote in the beginning, for terminology and notation not explained in this paper, we refer the reader to [21]. The connectivity, the independence number and the minimum degree of a graph G are denoted by $\kappa(G)$, $\alpha(G)$ and $\delta(G)$, respectively. We say that G is trivial if $|V(G)| = 1$. The degree of a vertex x in G is denoted by $\deg_G x$. For two vertex-disjoint graphs G and H, we write $G + H$ for the join of G and H. For a path $P = x_0x_1 \cdots x_m$, the subpath $x_ix_{i+1} \cdots x_j (i \leq j)$ is denoted by $x_i \overrightarrow{P} x_j$, and we write $x_j \overleftarrow{P} x_i$ for its reverse $x_jx_{j-1} \cdots x_i$.

2. Hamiltonian properties of k-connected $\{K_{1,k+1}, P_4\}$-free graphs

In this section, we study Hamiltonian properties of k-connected $\{K_{1,k+1}, P_4\}$-free graphs. In [8], the second author of this paper makes the following observations.

Theorem A (*[8]*)**.** *Let G be a connected noncomplete P_4-free graph, and let S be a smallest cutset of G. Then each vertex in S is adjacent to all vertices in $V(G) - S$.*

Theorem B (*[8]*)**.** *Let k be a positive integer and let G be a connected P_4-free graph. Then G is $K_{1,k+1}$-free if and only if $\alpha(G) \leq k$.*

He also points out that Theorem A has been proved implicitly by Faudree et al. [10] and that Theorem B is an immediate corollary of Theorem A.

We also use the following well-known theorem.

Theorem C (*Chvátal and Erdős [7]*)**.** *Let G be a 2-connected graph.*

(1) *If $\alpha(G) \leq \kappa(G)$, then G is Hamiltonian.*
(2) *If $\alpha(G) \leq \kappa(G) - 1$, then G is Hamiltonian-connected.*

By combining Theorems B and C(1), we immediately obtain the following corollary.

Corollary 1. *For $k \geq 2$, $\{K_{1,k+1}, P_4\}$ is a strong k-forbidden pair.*

However, k-connected $\{K_{1,k+1}, P_4\}$-free graphs satisfy stronger Hamiltonian properties.

Theorem 2. *For $k \geq 2$, every k-connected $\{K_{1,k+1}, P_4\}$-free graph is either pancyclic or isomorphic to $K_{k,k}$.*

Before proving the above theorem, we make one simple observation. A path-factor of a graph G is a spanning subgraph of G in which each component is a path.

Lemma 3. *A graph H of independence number at most k has a path-factor with at most k components. Moreover, if $|V(H)| \geq k$, then H has a path-factor with exactly k components.*

Proof. If $k = 1$, then H is a complete graph and the theorem trivially holds. Thus, we may assume $k \geq 2$. Let G be the graph obtained by the join of H and a complete graph of order k. Then G is k-connected and $\alpha(G) \leq k$. Since $k \geq 2$, G has a Hamiltonian cycle C by Theorem C(1). Then $V(C) \cap V(H)$ induces a path-factor with at most k components.

If $|V(H)| \geq k$ and the obtained path-factor has less than k components, it has a non-trivial component. By deleting an edge in this component, we can increase the number of components. We can continue this edge-deletion until the number of components reaches k. □

We also use the following theorem in the proof of Theorem 2.

Theorem D (*Bondy [2]*). *For $n \geq 3$, a graph of order n and minimum degree at least $\frac{1}{2}n$ is either pancyclic or isomorphic to $K_{k,k}$, where $k = \frac{1}{2}n$.*

Proof of Theorem 2. We proceed by induction on $|V(G)|$. If $|V(G)| = k+1$, then G is a complete graph of order $k+1$ and the theorem follows immediately. Thus, we may assume $|V(G)| \geq k+2$.

Suppose G is $(k+1)$-connected. Let x be a vertex of smallest degree in G, and let $G' = G - x$. Then G' is k-connected and $\{K_{1,k+1}, P_4\}$-free. If $G' \simeq K_{k,k}$, then since $\delta(G) \geq k+1$, every vertex in G' is adjacent with x. This contradicts the minimality of $\deg_G x$. Therefore, G' is not isomorphic to $K_{k,k}$. Then by the induction hypothesis, G' is pancyclic. Since G is Hamiltonian by Corollary 1, G is also pancyclic. Therefore, we may now assume that the connectivity of G is exactly k. If $|V(G)| \leq 2k$, then $\delta(G) \geq \kappa(G) = k \geq \frac{|V(G)|}{2}$, and hence G is either pancyclic or isomorphic to $K_{k,k}$ by Theorem D. Therefore, we may further assume $|V(G)| \geq 2k+1$.

Let $S = \{z_1, \ldots, z_k\}$ be a smallest cutset of G and let $H = G - S$. By Theorem A, G is the join of H and $G[S]$. By Theorem B, $\alpha(G) \leq k$ and hence $\alpha(H) \leq k$. Moreover, since $|V(G)| \geq 2k+1$, $|V(H)| \geq k+1$. Then by Lemma 3, H has a path-factor F with exactly k components, and at least one of its components is nontrivial.

Let $P_1, \ldots, P_k$ be the components of F, each of which is a path. We may assume $|V(P_1)| \geq 2$. Let x_i and y_i be the first and the last vertices of P_i. Note that $x_1 \neq y_1$, but possibly $x_i = y_i$ for some $i \geq 2$. Since $|V(P_1)| \geq 2$, we can define the successor x_1^+ of x_1 in P_1. Then $x_1x_1^+z_1x_1$ is a cycle of order three.

For each integer l with $3 \leq l \leq |V(P_1)|+1$, we can choose a vertex $v \in V(P_1)$ with $|V(x_1\overrightarrow{P_1}v)| = l-1$, and then $x_1\overrightarrow{P_1}vz_1x_1$ is a cycle of order l. Let $C_1 = x_1\overrightarrow{P_1}y_1z_1x_1$. Then C_1 is a cycle with $V(C_1) = V(P_1) \cup \{z_1\}$ and $\{z_1x_1, x_1x_1^+, y_1z_1\} \subset E(C_1)$.

Suppose for an integer m with $1 \leq m < k$, G has a cycle of order l for every integer l with $3 \leq l \leq \sum_{i=1}^{m}(|V(P_i)|+1)$. Also suppose G has a cycle C_m with $V(C_m) = \bigcup_{i=1}^{m} V(P_i) \cup \{z_1, \ldots, z_m\}$ and $\{z_1x_1, x_1x_1^+, y_mz_m\} \subset E(C_m)$. By replacing the edge y_mz_m with the path $y_mz_{m+1}x_{m+1}z_m$, we obtain a cycle C' of order $|V(C_m)|+2$. Moreover, by replacing the subpath $z_1x_1x_1^+$ in C' with the edge $z_1x_1^+$, we obtain a cycle of order $|V(C')| - 1 = |V(C_m)| + 1$. For each l with $2 \leq l \leq |V(P_{m+1})|+1$, we can take a vertex v in $V(P_{m+1})$ with $|V(x_{m+1}\overrightarrow{P_{m+1}}v)| = l-1$. Then by replacing the edge y_mz_m with the path $y_mz_{m+1}v\overleftarrow{P_{m+1}}x_{m+1}z_m$, we obtain a path of order $|V(C_m)|+l$. In particular, let C_{m+1} be the cycle obtained from C_m by replacing the edge y_mz_m with $y_mz_{m+1}y_{m+1}\overleftarrow{P_{m+1}}x_{m+1}z_m$. Then $V(C_{m+1}) = V(C_m) \cup V(P_{m+1}) \cup \{z_{m+1}\} = \bigcup_{i=1}^{m+1} V(P_i) \cup \{z_1, \ldots, z_{m+1}\}$ and $\{z_1x_1, x_1x_1^+, y_{m+1}z_{m+1}\} \subset E(C_{m+1})$.

By the construction, when we construct C_k, we obtain a cycle of order l for each l with $3 \leq l \leq |V(G)|$. □

While every k-connected $\{K_{1,k+1}, P_4\}$-free graph except for $K_{k,k}$ is pancyclic, there exist infinitely many k-connected $\{K_{1,k+1}, P_4\}$-free graphs which are not Hamiltonian-connected. For example, for each positive integer m, the graph $kK_m + kK_1$ is k-connected and $\{K_{1,k+1}, P_4\}$-free, but no Hamiltonian path exists joining a pair of vertices in kK_1. On the other hand, a k-connected $\{K_{1,k}, P_4\}$-free graph is Hamiltonian-connected by Theorem C(2) since its independence number is at most $k-1$ by Theorem B.

3. Forbidden pairs and stars

In this section, we prove that every k-forbidden pair contains a star with at most $k+1$ leaves. But before that, we prepare a k-connected graph of girth at least five, which we use in the proof.

Let n and k be two positive integers with $n \geq 2k+1$. The Kneser graph $K(n, k)$ is the graph whose vertices represent the k-subsets of $\{1, 2, \ldots, n\}$ and where two vertices are adjacent if they correspond to disjoint subsets. Clearly, $K(n, k)$ has $\binom{n}{k}$ vertices and it is a regular graph of degree $\binom{n-k}{k}$. Moreover, $K(n, k)$ is edge-transitive and hence the connectivity of $K(n, k)$ coincides with its degree $\binom{n-k}{k}$ (see [16], for example). In particular, $K(2k-1, k-1)$ is a k-connected k-regular graph. Moreover, if $k \geq 3$, its girth is at least five.

We need another class of graphs, which was introduced in [19].

Theorem E (*[19]*). *For each positive integer m, n and k with $n \geq k+1$, there exists a k-connected unbalanced bipartite graph of maximum degree exactly n and order at least m.*

Now we prove the main theorem in this section.

Theorem 4. *Let k be an integer with $k \geq 2$ and let F_1 and F_2 be connected graphs of order at least three. If $\{F_1, F_2\}$ is a k-forbidden pair, then either F_1 or F_2 is isomorphic to $K_{1,l}$, where $l \leq k+1$.*

Proof. Let $\{F_1, F_2\}$ be a k-forbidden pair. Then there exists a positive integer n_0 such that every k-connected $\{F_1, F_2\}$-free graph of order at least n_0 is Hamiltonian. We may assume $n_0 \geq k+1$.

Assume neither F_1 nor F_2 is isomorphic to $K_{1,l}$ for any $l \leq k+1$. Since K_{n_0,n_0+1} is a k-connected non-Hamiltonian graph of order at least n_0, it contains F_1 or F_2 as an induced subgraph. By symmetry, we may assume that F_1 is an induced subgraph of K_{n_0,n_0+1}. Then F_1 itself is a complete bipartite graph. In particular, F_1 does not contain a triangle. Moreover, F_1 either is a star with at least $k+2$ leaves or contains C_4.

Let $G_1 = K_k + (k+1)K_{n_0}$. Then G_1 is a k-connected non-Hamiltonian graph of order at least n_0. Then G_1 contains F_1 or F_2 as an induced subgraph. Note that every connected induced subgraph of G_1 either contains a triangle or is isomorphic to $K_{1,l}$ for some $l \le k+1$. Therefore, F_1 is not an induced subgraph of G_1 and hence G_1 contains F_2 as an induced subgraph. This implies that F_2 contains a triangle.

Let $H_1, H_2, \ldots, H_{k+1}$ be copies of the Kneser graph $K(2n_0 - 1, n_0 - 1)$. Since $n_0 \ge k+1$, each H_i is k-connected and of order at least n_0. For each i, $1 \le i \le k+1$, we take k distinct vertices $y_{i,1}, \ldots, y_{i,k}$ of H_i. We then introduce k new vertices $x_1, \ldots, x_k$ and join x_j and $y_{i,j}$ for each i and j with $1 \le i \le k+1$ and $1 \le j \le k$. Let G_2 be the resulting graph.

It is not difficult to see that G_2 is a k-connected graph of girth at least five and order greater than n_0. Moreover, G_2 is not 1-tough and hence it is not Hamiltonian. Therefore, G_2 contains either F_1 or F_2 as an induced subgraph. However, since G_2 does not contain a triangle, F_2 is not an induced subgraph of G_2, and hence G_2 contains F_1 as an induced subgraph. Moreover, since G_2 does not contain C_4, F_1 is isomorphic to a star $K_{1,l}$, where $l \ge k+2$.

Theorem E guarantees the existence of a k-connected unbalanced bipartite graph G_3 of maximum degree exactly $k+1$ and order at least n_0. Then G_3 is not Hamiltonian. Since the maximum degree of G_3 is $k+1$, it does not contain F_1 as an induced subgraph. And since G_3 does not contain a triangle, F_2 is not an induced subgraph of G_3. Therefore, G_3 is a k-connected $\{F_1, F_2\}$-free non-Hamiltonian graph of order greater than n_0. This is a contradiction, and the theorem follows. □

4. Concluding remarks

In this paper, we have investigated pairs of forbidden subgraphs which force Hamiltonicity of k-connected graphs, possibly with a finite number of exceptions. While all the 2-forbidden pairs contain $K_{1,3}$, as has been proved in [11], we have seen that for $k \ge 3$, there exists a strong k-forbidden pair not containing $K_{1,3}$. On the other hand, every k-forbidden pair contains $K_{1,l}$ for some $l \le k+1$.

According to [11], the set of 2-forbidden pairs is finite. It may suggest that the set of all the k-forbidden pairs is finite for every $k \ge 2$. But we need some preparation to make it a formal conjecture.

Matthews and Sumner [17] made the following conjecture.

Conjecture F (*[17]*)**.** *Every 4-connected claw-free graph is Hamiltonian.*

As a partial solution, Ryjáček [20] proved that every 7-connected claw-free graph is Hamiltonian. This result implies that for every connected graph H, $\{K_{1,3}, H\}$ is a 7-forbidden pair. In particular, there exist infinitely many strong 7-forbidden pairs. They may even be strong 4-forbidden pairs if Conjecture F is true.

In order to avoid set-theoretic ambiguity, we consider graphs whose vertices are natural numbers. Let $\mathcal{G}$ be the set of all the finite connected graphs of order at least three. Let $\boldsymbol{F}'_k = \{\mathcal{F} \subset \mathcal{G}:$ there exists a natural number n_0 such that every k-connected $\mathcal{F}$-free graph of order at least n_0 is Hamiltonian $\}$. By the definition, for each $\mathcal{F} \in \boldsymbol{F}'_k$ and $\mathcal{H} \subset \mathcal{G}$, $\mathcal{F} \cup \mathcal{H} \in \boldsymbol{F}'_k$. Let $\boldsymbol{F}_k$ be the set of all the minimal elements in terms of inclusion,

$$\boldsymbol{F}_k = \{\mathcal{F} \in \boldsymbol{F}'_k : \mathcal{F}' \notin \boldsymbol{F}'_k \text{ for any } \mathcal{F}' \subsetneq \mathcal{F}\}.$$

In other words, $\boldsymbol{F}_k$ is the subfamily of $\boldsymbol{F}'_k$ consisting of non-redundant members.

For a positive integer p, define $\boldsymbol{F}^p_k$ by:

$$\boldsymbol{F}^p_k = \{\mathcal{F} \in \boldsymbol{F}_k : |\mathcal{F}| \le p\}.$$

The result of Faudree and Gould [11] determines $\boldsymbol{F}^2_2$, and Corollary 1 and Theorem 4 claim $\{K_{1,k+1}, P_4\} \in \boldsymbol{F}^2_k - \boldsymbol{F}^2_{k-1}$ for each $k \ge 3$. The result of Ryjáček claims $\{K_{1,3}\} \in \boldsymbol{F}^1_7$.

Now we can state our conjecture in a formal way.

Conjecture 1. *For each integer k with $k \ge 2$, $\boldsymbol{F}^2_k$ is a finite set.*

The complete characterization of $\boldsymbol{F}^2_4$ involves Conjecture F and seems to be difficult.

We make one more comment before closing this section. In [15], it is shown that there is a natural partial order in $\boldsymbol{F}_k$. For $\mathcal{F}_1, \mathcal{F}_2 \subset \mathcal{G}$, we write $\mathcal{F}_1 \le \mathcal{F}_2$ if for each $F \in \mathcal{F}_2$, there exists $F' \in \mathcal{F}_1$ such that F' is an induced subgraph of F. Trivially, if $\mathcal{F}_1 \le \mathcal{F}_2$, then every $\mathcal{F}_1$-free graph is $\mathcal{F}_2$-free. Moreover, $\le$ is a reflexive and transitive binary relation in $2^{\mathcal{G}}$. Though it is not anti-symmetric in $\boldsymbol{F}'_k$, it is anti-symmetric in $\boldsymbol{F}_k$ and hence $(\boldsymbol{F}_k, \le)$ is a partially ordered set.

We make the following observation.

Theorem 5. *If $\mathcal{F} \in \boldsymbol{F}^p_k$, then $\{\mathcal{F}' \in \boldsymbol{F}_k : \mathcal{F}' \le \mathcal{F}\} \subset \boldsymbol{F}^p_k$.*

Proof. Let $\mathcal{F} = \{F_1, F_2, \ldots, F_r\}$, where $r \le p$. Let $\mathcal{F}' \in \boldsymbol{F}_k$ and $\mathcal{F}' \le \mathcal{F}$. Then for each i with $1 \le i \le r$, there exists $F'_i \in \mathcal{F}'$ such that F'_i is an induced subgraph of F_i. Let $\mathcal{F}'' = \{F'_1, F'_2, \ldots, F'_r\} \subset \mathcal{F}'$. Then $\mathcal{F}'' \le \mathcal{F}$. Since $\mathcal{F} \in \boldsymbol{F}^p_k \subset \boldsymbol{F}_k$, there exists a positive integer n_0 such that every k-connected $\mathcal{F}$-free graph of order at least n_0 is Hamiltonian. Then every k-connected $\mathcal{F}''$-free graph of order at least n_0 is $\mathcal{F}$-free and hence it is also Hamiltonian. This implies $\mathcal{F}'' \in \boldsymbol{F}'_k$. Since $\mathcal{F}'' \subset \mathcal{F}'$ and $\mathcal{F}' \in \boldsymbol{F}_k$, we have $\mathcal{F}' = \mathcal{F}''$, which implies $\mathcal{F}' \in \boldsymbol{F}^p_k$. □

By this theorem, in order to determine $\boldsymbol{F}_k^p$, it is sufficient to characterize its maximal elements in terms of $\leq$. According to [11,1], $\boldsymbol{F}_2^2$ has five maximal elements and four of them are strong 2-forbidden pairs.

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