# Two extensions of Ramsey's theorem

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Ramsey's theorem, in the version of Erdős and Szekeres, states that every 2-coloring of the edges of the complete graph on $\{1, 2, \ldots, n\}$ contains a monochromatic clique of order $\frac{1}{2}\log n$. In this talk, we consider two well-studied extensions of Ramsey's theorem.

Improving a result of Rödl, we show that there is a constant $c > 0$ such that every 2-coloring of the edges of the complete graph on $\{2, 3, ..., n\}$ contains a monochromatic clique $S$ for which the sum of $1/\log i$ over all vertices $i \in S$ is at least $c \log\log\log n$. This is tight up to the constant factor $c$ and answers a question of Erdős from 1981.

Motivated by a problem in model theory, Vaananen asked whether for every $k$ there is an $n$ such that every graph on $\{1, \ldots, n\}$ contains a homogeneous set $a_1 < \ldots < a_k$ such that the differences $a_{i+1} - a_i$ satisfy a prescribed order relation. Alon and independently Pach answered this question affirmatively. Alon further conjectured that the true growth rate should be exponential in $k$. Shelah proved that $n$ is at most double-exponential in $k$. We make further progress towards Alon's conjecture, obtaining an upper bound on $n$ which is exponential in a power of $k$.

The proofs of the above results use a powerful probabilistic technique known as dependent random choice with additional combinatorial methods. Joint work with David Conlon and Benny Sudakov.