Appendix

This appendix contains some of the definitions and formulas that were used within the text.

1. **The Binomial Coefficients:**

$$\binom{m}{n} = \begin{cases} \dfrac{m!}{n!(m-n)!} & \text{if } n \neq 0,\ m \geq n, \\ 1 & \text{if } n = 0,\ m \geq n, \\ 0 & \text{if } m < n. \end{cases}$$

2. **Convex Function:** If

$$f\left(\frac{x+y}{2}\right) \leq \frac{f(x)+f(y)}{2}$$

on an interval $[a, b]$, then *f* is *convex* on that interval.

3. If *f* is convex on $[a, b]$, then

$$f\left(\frac{\sum_1^n x_i}{n}\right) \leq \frac{\sum_1^n f(x_i)}{n}$$

on the interval $[a, b]$.

4. **Jensen's Inequality:** If $p_i > 0$ $(i = 1, \ldots, n)$ and *f* is convex on (a, b), then

$$f\left(\frac{\sum_1^n p_i x_i}{\sum_1^n p_i}\right) \leq \frac{\sum_1^i p_i f(x_i)}{\sum_1^n p_i}$$

on the interval (a, b).

5. **Cauchy's Inequality** If the numbers $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ are nonnegative, then

$$\sum_1^n a_i b_i \le \left(\left(\sum_1^n a_i^2 \right) \left(\sum_1^n b_i^2 \right) \right)^{1/2}.$$

6. **Markov's Theorem** If $X \ge 0$ and $t > 0$, then

$$Pr(X \ge t) \le \frac{E(X)}{t}.$$

7. **Stirling's Formula** The factorial function can be estimated as

$$n! = [1 + o(1)]\sqrt{2\pi n}\left(\frac{n}{e}\right)^n.$$

8. **Chebyshev's Inequality** For $t > 0$,

$$Pr(|X - E(X)| \ge t) \le \frac{V(X)}{t^2}.$$

9. **Big Oh Notation** For sequences $\{a_n\}$ and $\{b_n\}$ of real numbers we say that: $a_n = O(b_n)$ if there exist constants K and N such that $|a_n| \le Kb_n$ for all $n > N$.

   Note that $a_n = O(1)$ means $\{a_n\}$ is bounded.

10. **Little Oh Notation** We say that $a_n = o(b_n)$ if the exists a sequence $\{k_n\}$ of positive terms such that $k_n \to 0$ and a constant N so that $|a_n| \le k_n b_n$ for all $n > N$.

    Note that $a_n = o(1)$ if $a_n \to 0$.

11. **Asymptotic Equivalence** By $a_n = b_n(1 + o(1))$ we mean that a_n and b_n are asymtotically equivalent (denoted $a_n \sim b_n$).

    Note that definitions similar to 7-9 hold for arbitrary functions: For example, if $\lim_{n\to\infty} \frac{f(n)}{g(n)} \to 0$ then we write $f(n) = o(g(n))$; and $f \sim g$ if $f(n) = (1 + o(1))g(n)$, that is, $\lim_{n\to\infty} \frac{f(n)}{g(n)} \to 1$.