A Proof of an Asymptotic Version of a Conjecture by Enomoto and Ota.

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In 2000, Enomoto and Ota [1] stated the following conjecture:

Conjecture 1 *Let G be a graph of order n, and $n_1, n_2, ..., n_t$ be positive integers with $\sum_{i=1}^{t} n_i = n$. If $\sigma_2(G) \geq n + t - 1$, then for any t distinct vertices $x_1, x_2, ..., x_t$ in G, there exist vertex disjoint paths $P_1, P_2, ..., P_t$ such that $|V(P_i)| = n_i$ for $1 \leq i \leq t$.*

The cases where $t = 1, 2$ follow from Theorems by Ore [2, 3]. Enomoto and Ota proved the conjecture for $t = 3$ and when almost all the n_i are less than six. We prove the following asymptotic version of this conjecture:

Theorem 1 *Suppose we are given real numbers $\eta_1, \eta_2, ..., \eta_t > 0$ with $\sum \eta_i = 1$ and $0 < \epsilon < \min\{\frac{\eta_i}{2}\}$. Then for n sufficiently large, every graph G on n vertices satisfying $\sigma_2 \geq n + t - 1$ contains t vertex disjoint paths of lengths l_i with $(\eta_i - \epsilon)n < l_i < (\eta_i + \epsilon)n$ starting at any set of t predetermined vertices x_i.*

Our proof includes three main steps. The first creates a spanning collection of vertex disjoint paths starting at the chosen vertices. The second step moves vertices from paths which are too long to paths which are too short if certain conditions are satisfied. When these conditions are not satisfied, the graph had some structure which allows us to build the desired path system directly.

References

[1] Hikoe Enomoto and Katsuhiro Ota. Partitions of a graph into paths with prescribed endvertices and lengths. *J. Graph Theory*, 34(2):163–169, 2000.

[2] Oystein Ore. Note on Hamilton circuits. *Amer. Math. Monthly*, 67:55, 1960.

[3] Oystein Ore. Hamilton connected graphs. *J. Math. Pures Appl. (9)*, 42:21–27, 1963.