ON IRREDUCIBLE NO-HOLE L(2,1)-COLORING OF TREES

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An L(2,1)-coloring of a graph is a coloring of its vertices so that the colors of adjacent vertices differ by at least two and the colors of vertices with a common neighbor differ by at least one. The span of a graph \( G \), denoted \( \lambda(G) \), is the smallest number \( \lambda \) such that there is an L(2,1)-coloring of \( G \) using the integers 0, \ldots, \( \lambda \). Such colorings were first studied by Griggs and Yeh where they show that \( \Delta + 1 \leq \lambda(T) \leq \Delta + 2 \) for all trees \( T \).

Recently, Laskar and Villalpando defined the notion of irreducible no-hole colorability. An L(2,1)-coloring \( f \) is said to be no-hole provided all colors 0, \ldots, \( k \) are used, for some \( k \); it is irreducible provided that decreasing the color of any vertex results in a coloring that is no longer an L(2,1)-coloring. The inh-span of an graph \( G \), denoted \( \lambda_{inh}(G) \), is the smallest number \( \lambda \) such that there is an irreducible no-hole coloring of \( G \) using the integers 0, \ldots, \( \lambda \). Laskar and Villalpando proved that if \( T \) is a tree that is not a star, then \( \Delta + 1 \leq \lambda_{inh}(T) \leq \Delta + 2 \).

In this talk, we show that for most trees \( T \), the inh-span of \( T \) is equal to its span, that is, \( \lambda_{inh}(T) = \lambda(T) \).