Nordhaus-Gaddum Bounds for $k$-Domination in Graphs

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Abstract

A $k$-dominating set of a graph $G$ is a set $S$ of vertices of $G$ such that every vertex outside of $S$ has $k$ neighbors in $S$. The $k$-domination number of $G$, written $\gamma_k(G)$, is the size of the smallest $k$-dominating set in $G$. In this paper, we derive sharp upper and lower bounds on $\gamma_k(G) + \gamma_k(\overline{G})$ and $\gamma_k(G)\gamma_k(\overline{G})$, where $\overline{G}$ is the complement of $G$. We use the results for $k = 2$ to prove a conjecture of Alon, Balogh, Bollobás, and Szabó on game domination numbers.