Survey of Saturation Numbers

Ralph J. Faudree Department of Mathematical Sciences
University of Memphis
Memphis, TN 38152

A graph G is an F-saturated graph if G does not contain F as a subgraph, but $G \cup \{e\}$ contains a copy of F for any edge e not in G. The *saturation number* of F, denoted by $sat(F, n)$, is the minimum number of edges in an F-saturated graph G of order n, and $SAT(F, n)$ denotes the extremal class of F-saturated graphs of order n with $sat(F, n)$ edges. A survey of some of the classical results as well as some recent unpublished results on saturation numbers will be presented. The focus will be on results in which the 1986 Kászonyi and Tuza upper bound for saturation numbers gives the exact result, or gives a close approximation to the exact result. The Kászonyi and Tuza result verifies that $sat(n, F)$ is linear in n for any graph F. Let $\alpha(F)$ be the independence number of F, and define the following two parameters:

$$u = u(F) = |V(F)| - \alpha(F) - 1,$$

and

$$d = d(F) = \min\{|E(F')| \ : \ F' \text{ is induced by } S \cup x\},$$

where S is a maximal independent set and $x \in V(F) - S$. Then

$$sat(n, F) \leq un + \lfloor (d-1)(n-u)/2 \rfloor - \binom{u+1}{2},$$

and the graph $G = K_u + H$, where H is a nearly $(d-1)$-regular graph of order $n - u$ (where nearly $(d-1)$-regular means every vertex has degree $d - 1$ except for possibly one vertex) is a candidate to be in $Sat(n, F)$. Open questions will be presented.