$\mathcal{R}$-parameters and $\mathcal{R}$-chromatic problems

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For any graph G, a collection $\mathcal{R} = \{R_1, R_2, \ldots, R_t\}$ of subsets of the vertex set $V(G)$ can be selected. Two such common choices for $\mathcal{R}$ are $\mathcal{R} = E(G)$, the collection of edges, and $\mathcal{R} = \{N(v_1), N(v_2), \ldots, N(v_n)\}$, the collection of open neighborhoods. A number of general $\mathcal{R}$-parameters will be defined, many of which are instances of well studied parameters (such as domination and independence) as well as instances of previously undocumented parameters.

$\mathcal{R}$-chromatic problems involve these same $\mathcal{R}$ collections, where the $\mathcal{R}$-chromatic number $\chi_{\mathcal{R}}(G)$ can be defined to be the minimum number k of sets $C_1, C_2, \ldots, C_k$ that partition $V(G)$ such that, for each R_i, no C_j contains R_i. In particular, we consider the case where $\mathcal{R}$ is the collection of all open neighborhoods and consider $\chi_{(2)}(G)$, where $\chi_{(2)}(G)$ is defined to be the minimum number k of sets $C_1, C_2, \ldots, C_k$ that partition $V(G)$ such that no C_j contains any open neighborhood $N(v)$.