S. B. Rao's Well-Quasi-Ordering Conjecture for Degree Sequence of Finite Graphs

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Let $D = (d_1, \ldots, d_n)$ be a finite sequence of natural numbers in non-descending order and $R[D]$ be the set of all non-isomorphic graphs G on n vertices with D their degree-sequence. Let P be a set of all degree sequences D with $R[D]$ non-empty and let D_1, $D_2 \in P$. We say that $D_1 \leq D_2$ if there exist $H \in R[D_1]$ and $G \in R[D_2]$ such that H is a vertex induced subgraph of G. It can be easily checked that "$\leq$" defines a partial ordering on P. In 1980/81 S. B. Rao conjectured that $\leq$ is a well-quasi-order; i.e., for any infinite sequence D_1, D_2, $\ldots$ in P, there exist indices $i < j$ such that $D_i \leq D_j$. The truth of Rao's conjecture does not seem to follow from the deep Graph Minor Theorem of Robertson and Seymour. We prove Rao's conjecture for claw-free degree sequences.

This is joint work with Neil Robertson.