Class 111, Calculus
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Remarks to common mistakes

1. It is important that you check if you can apply some rules/ propositions. It is NOT enough if some of the assumptions are given - **every assumption has to be given!** If that is not the case you may end up with a wrong conclusion! And you have to verify in homework assignments, quizzes and other exams why you can apply a rule!

Example: Consider

$$\lim_{x\to 3} \frac{x^4-81}{x^2-9}$$

WRONG!!!:
A lot of you tend to apply the quotient rule in this case. If you do so, you get the following conclusion:

$$\begin{aligned}\lim_{x\to 3} \frac{x^4-81}{x^2-9} &= \frac{\lim_{x\to 3} x^4-81}{\lim_{x\to 3} x^2-9}\\ &= \ldots\\ &= \infty\end{aligned}$$

RIGHT:

$$\begin{aligned}\lim_{x\to 3} \frac{x^4-81}{x^2-9} &= \lim_{x\to 3} \frac{(x^2-9)(x^2+9)}{x^2-9}\\ &= \lim_{x\to 3} x^2+9\\ &= 3+9\\ &= 12\end{aligned}$$

Note, that you CANNOT (!!!!!) apply the quotient rule if the limit of the denominator goes to zero! (Again: There are similar examples for the product rule, Squeeze theorem,... - if one of the assumptions for the theorem/ rule is not given, you cannot apply the rule. If you do, it is very likely that you get to a wrong conclusion!)

2. A lot of you seem to be confused between taking the limit of a function and deciding if a function is actually continuous at some point. Note that if you consider

$$\lim_{x\to a} f(x)$$

a does not need to be in the domain of $f(x)$. The function doesn't even have to be properly defined there. An example for that would be

$$\lim_{x\to\infty} \frac{1}{x}.$$

Clearly, you cannot plug-in $x=\infty$ into the function, whereas the limit is well-defined there.

Now, let's take a look at continuity:
By the definition you have to check for a function to be continuous if

$$\lim_{x \to a} f(x) = f(a).$$

Hence, you have to be able to plug-in the number a on the right hand side of the definition. Therefore, $f(a)$ needs to be well-defined, meaning that a needs also to be in the domain of the function.