Class 111, Calculus
Instructor: Silke Gehrke

Summary

Relates Rates

Algorithm to get the solution:

1. Understand the problem and draw a diagram
2. Write down given information using variables
3. Write down what is unknown
4. Make sure you use the same units everywhere
5. Relate given and unknown in an equation
6. Use given information in order to take the derivative on both sides of equation of previous step (chain rule!)
7. Plug in given information to obtain solution

Functions and their extreme values

Def. global max/ (min)

A function f has an absolute (global) maximum (minimum) at a number c, if $f(c) \geq f(x)$ ($f(c) \leq f(x)$)for all x in the domain of f. These values are calles the **extreme values** of a function.

Def. local max (min)

A function f has a local maximum (minimum) at a number c in the domain of f, if $f(c) \geq f(x)$ ($f(c) \leq f(x)$) for $x \in (c-a, c+a), a > 0, a$small i.e. for all x in some open intervall containing c.

The Extreme Value Theorem

If f is a **continuous** function on a closed intervall $[a, b]$, then f attains an absoulte max. value $f(c)$ and an absolute min. value $f(d)$, where c and d are in $[a, b]$.

Fermat's Theorem

If a function f has a local min./max at a point c and if f is differentiable, then $f'(c) = 0$.

Def. critical number

A critical number of a function f is a number c in the domain od f s.t. either $f'(c) = 0$ or $f'(c)$ does not exist.

Fermat's Theorem rewritten

If f has a local max./ min. at c, then c is a critical number of f.

Algorithm how to find absolute min/max

Given: continuous function on $[a, b]$

1. Find critical numbers on $[a, b]$. (i.e. find where $f'(x) = 0$ or where it does not exist.
2. Find $f(a)$ and $f(b)$.
3. The largest/smallest value of step 1 and step 2 is the absolute max/ min.

Rolle's Theorem

Let f be a function $f : [a, b] \to \mathbb{R}$ with the properties

1. f is continuous on $[a, b]$.
2. f is differentiable on (a, b)
3. $f(a) = f(b)$

Then there is a number $c \in (a, b)$ s.t. $f'(c) = 0$.

An application of Rolle's Theorem is for example to show, that an equation has exactly one solution. Then we first show that there exists at least a solution by applying the Intermediate Value Theorem and in the second step we show that there is exactly one. To do that, we assume by contradiction, that there is more than one solution.

The Mean Value Theorem

Let f be a function with the properties that f is continuous on $[a, b]$ and differentiable on (a, b). Then there is a number c in (a, b) s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$ or equivalently $f'(c)(b - a) = f(b) - f(a)$.

Theorem when f is a constant function

If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

Fact:

If $f'(x) > 0$ on an interval (a, b) then f is increasing on that interval. If $f'(x) < 0$ on an interval (a, b) then f is decreasing on that interval.

The first derivative test

Suppose c is a critical number of a differentiable function f. Then

1. If f' changes from + to - at c then f has a **local** maximum at c.
2. If f' changes from - to + at c then f has a **local** minimum at c.
3. If f' does not change sign at c then f has neither a local min nor a local max at c.

Definition concave, convex

If the graph f lies above all its tangents on an interval I, then f is called convex (=concave upward) on I. If the graph f lies below all its tangents on an interval I, then f is called concave (=concave downward) on I.

Concativity Test

If $f''(x) > 0$ for all x in (a, b), then the graph of f is convex on $[a, b]$.

If $f''(x) < 0$ for all x in (a, b), then the graph of f is concave on $[a, b]$.

Definition inflection point

Let f be a continuous function, then an inflection point of f is where f changes from convex to concave or vice versa.

The second derivative test

Suppose $f''(x)$ is continuous near c. Then if $f'(c) = 0$ and $f''(c) > 0$, then c is a local minimum of f.

If $f'(c) = 0$ and $f''(c) < 0$, then c is a local maximum of f.