Esther Beneish

*Failure of Krull-Schmidt for invertible lattices over a discrete valuation ring*

Let $G$ be a finite group and let $RG$ be with coefficients in a Dedekind domain $R$. An $RG$-lattice $M$ is defined to be a finitely generated $R$-torsion-free $RG$-module. $M$ is said to be a permutation lattice if it is $R$-free and has an $R$-basis permuted by $G$. $M$ is said to be an invertible or a permutation projective lattice, if it is a direct summand of a permutation lattice. We exhibit a category of invertible lattices over a discrete valuation ring, for which the Krull-Schmidt Theorem fails. The question of existence of a such category arose in the study of the problem of the uniqueness of a direct sum decomposition of the motive of a projective homogeneous variety into indecomposable objects in the category of Chow motives. This category contains a subcategory equivalent to the category of invertible lattices for a certain finite group. Failure of Krull-Schmidt for this subcategory implies failure of uniqueness of direct sum decompositions for the motives.

Vladimir Chernousov

Jean-Louis Colliot-Thélène

Philippe Gille

*Resolving $G$-torsors under abelian extensions*

This is a report on joint work with V. Chernousov and Z. Reichstein. We investigate the question of reducing $G$-torsors to finite subgroups for reductive group schemes over rings.

Stefan Gille
Let $k$ be a global field of characteristic not 2. Let $C$ be a geometrically irreducible nonsingular projective curve over $k$ and $K = k(C)$ be the function field of $C$ over $k$. For each prime spot $p$ on $k$, let $\hat{k}_p$ denote the corresponding completion of $k$ and $\hat{k}_p(C)$ be the function field of $C \times_k \hat{k}_p$. In this paper, we consider the affine curve of the form $y^2 = f(x)$ where $f(x)$ is an irreducible quartic curve over $k$ and explicitly describe all the constant classes (coming from $\text{Br}(k)$) in the kernel of the map

$$h: \text{Br}(K) \to \prod_p \text{Br}\left(\hat{k}_p(C)\right),$$

where $p$ ranges over all the prime spots of $k$.

Katrin Becher has recently found a surprisingly elementary proof of the Pfister Factor Conjecture. This conjecture originally due to Shapiro has been of great interest as it allows equivalent formulations in seemingly disparate contexts, such as composition of quadratic forms or algebras with involution. We will mention some of these formulations and explain Becher’s proof which makes clever use of some well-known facts about function fields of conics and how quadratic forms and involutions behave when extending the base field to such a function field of a conic.
Nikita Karpenko

*Quadratic forms in characteristic 2 and algebraic cobordism*

Several results on quadratic forms over fields of characteristic not 2 are proved using Steenrod operations on Chow groups modulo 2. Construction of the Steenrod operations in characteristic 2 is not available. However, assuming resolution of singularities, one can get similar results in characteristic 2 using the theory of algebraic cobordism. This is a joint work in progress with Olivier Haution.

Max Knus

*From classical groups to étale algebras*

Following A. Weil, central simple algebras with involution are classified by the Galois cohomology of classical groups. The cohomology of the corresponding Weyl groups classify étale algebras with involution. Formal analogies lead to extending constructions for central simple algebras to étale algebras, one such construction being the Clifford algebra. The analogy is illustrated by low-dimensional examples. (Joint work with J.-P. Tignol.)

Daniel Krashen

*Splitting fields of separable algebras*

Given a field $F$, an étale extension $L/F$ and an Azumaya algebra $A/L$, one knows that there are extensions $E/F$ such that $A \otimes E$ is a split algebra over $L \otimes E$. In this talk we bound the degree of a minimal splitting field of this type from above and show that our bound is sharp in certain situations, even in the case where $L/F$ is a split extension. This gives in particular some generalizations of the classical fact that when the tensor product of two quaternion algebras is not a division algebra, the two quaternion algebras must share a common quadratic splitting field.

Nicole Lemire

*Galois module structure of Galois cohomology and applications*

For a cyclic $p$-extension of fields $E/F$ where $F$ contains a primitive $p$-th root of unity, we determine the $\mathbb{F}_p[\text{Gal}(E/F)]$-module structure of $H^m(G_E, \mathbb{F}_p)$ in terms of the field extension $E/F$. This is joint work with Jan Minac and John Swallow. We then apply this to determine restrictions on the group structure of an absolute Galois group $G_F$ (with Dave Benson) and to determine the cohomological dimension of the maximal pro-$p$ quotient $G_F(p)$ (with John Labute).
Max Lieblich

Eli Matzri

All dihedral division algebras of degree five are cyclic

A theorem of Rowen and Saltman says that every division algebra which is split by a dihedral extension of degree $2n$ of the center, $n$ odd, is in fact cyclic. The proof requires roots of unity of order $n$ in the center. We show that for $n = 5$, this assumption can be removed. It then follows from a work of Vishne that $5\text{Br}(F)$, the 5-torsion part of the Brauer group, is generated by cyclic algebras, generalizing a result of Merkurjev on the 2 and 3 torsion parts.

Kelly McKinnie

Galois subfields in generic abelian crossed products and indecomposable $p$-algebras

I will talk about conditions on a valued $p$-algebra under which the algebra has only inertial Galois subfields. Using this result one can show that non-cyclic generic abelian crossed product $p$-algebras defined by non-degenerate matrices are indecomposable $p$-algebras. To show that such abelian crossed products exist with prime exponent and arbitrarily large exponent we prove that the abelian crossed products of exponent $p$ and index $p^n$ $(p \neq 2)$, which are gotten by generically reducing the exponent, are defined by non-degenerate matrices.

Alexander Merkurjev

$R$-equivalence on 3-dimensional tori and zero-cycles

Let $X$ be a smooth proper model of an algebraic torus $T$. We prove that if $\dim T \leq 3$ then the map taking a rational point $t$ of $T$ to the zero cycle $[t] - [1]$ on $X$ yields an isomorphism between $T(F)/R$ and the Chow group of zero-dimensional algebraic cycles on $X$ of degree zero.

Holger Petersson

Composition algebras over commutative rings

Abstract TBA
Mélanie Raczek
Central simple algebras of degree 3 and ternary cubic forms

We use Galois cohomology to classify pairs \((A, V)\), where \(A\) is a central simple \(F\)-algebra of degree 3 and \(V\) is a 3-dimensional subspace of trace zero elements that is totally isotropic for the trace quadratic form. To each pair \((A, V)\) is associated a ternary cubic form \(V \to F: \xi \mapsto \xi^3\). We use the classification of the pairs \((A, V)\) to describe those cubic forms.

Zinovy Reichstein
Essential dimension

The essential dimension of an algebraic object (e.g., of an algebra, a quadratic form, an algebraic variety or a principal homogeneous space) is the minimal possible number of independent parameters required to define the underlying structure. In recent years this numerical invariant has been studied by a variety of algebraic, geometric and cohomological techniques. The goal of my talk is to give an introduction to this subject and to survey the latest developments.

Louis Rowen

David Saltman

Claus Schubert
Going Up of the \(u\)-Invariant over formally real fields

The (general) \(u\)-invariant of a field \(F\) of characteristic \(\neq 2\) is defined to be \(u(F) := \max\{\dim \varphi \mid \varphi\) anisotropic torsion form over \(F\}\) or \(\infty\) if no such maximum exists. Let \(F\) be a formally real field and assume \(F\) has finite reduced stability. We prove that \(u(F)\) is finite if and only if \(u(F(\sqrt{-1}))\) is finite. This implies in particular that for any finite extension \(K/F\), if \(u(F)\) is finite then \(u(K)\) is finite.
Venapally Suresh
*Bounding the symbols length over a function field of p-adic curves*

Let $k$ be a $p$-adic field and $K$ a finite extension of $k(t)$. Let $q$ be a prime not equal to $p$. Assume that $K$ contains all the $q$-th roots of unity. We show that every element in $H^2(K, \mu_q)$ is a sum of 2 symbols and every element in $H^3(K, \mu_l)$ is a symbol.

Jean-Pierre Tignol
*The discriminant of symplectic involutions*

The Rost invariant of symplectic groups can be used to define a degree 3 cohomological invariant of symplectic involutions on central simple algebras of degree $4n$ for $n \geq 2$. In degree 8, vanishing of the discriminant characterizes involutions that decompose into a tensor product of involutions on quaternion algebras. (Joint work with S. Garibaldi and R. Parimala.)

Burt Totaro
*Birational geometry of quadrics using Chow groups*

A fundamental problem of the theory of quadratic forms is to classify quadrics over an arbitrary field up to birational equivalence. For everywhere non-smooth quadrics in characteristic 2, we can solve some of the main problems about birational classification, which remain open for all other types of quadrics. As in Karpenko and Merkurjev’s theorem on the essential dimension of quadrics, the key is to study the Chow motives of quadrics (or, in more elementary terms, Chow groups of products of quadrics).

Uzi Vishne
Adrian Wadsworth

*Value functions and associated graded rings for semisimple algebras*

This is joint work with J.-P. Tignol. We introduce a type of value function \( y \) called a *gauge* on a finite-dimensional semisimple algebra \( A \) over a field \( F \) with valuation \( v \). The filtration on \( A \) induced by \( y \) yields an associated graded ring \( gr_y(A) \) which is a graded algebra over the graded field \( gr_v(F) \).

Key requirements for \( y \) to be a gauge are that \( gr_y(A) \) be graded semisimple and that \( \dim_{gr_v(F)}(gr_y(A)) = \dim_F(A) \). It is shown that gauges behave well with respect to scalar extensions and tensor products. When \( v \) is Henselian and \( A \) is central simple over \( F \), it is shown that \( gr_y(A) \) is simple and graded-Brauer-equivalent to \( gr_w(D) \) where \( D \) is the division algebra Brauer-equivalent to \( A \) and \( w \) is the valuation on \( D \) extending \( v \) on \( F \). The utility of having a good notion of value function for central simple algebras, not just division algebras, and with good functorial properties, is demonstrated by giving new and greatly simplified proofs of some difficult earlier results on valued division algebras.

Olivier Wittenberg

*Albanese torsors and the elementary obstruction*

Let \( X \) be a smooth proper geometrically connected variety over a field \( k \). Colliot-Thélène and Sansuc introduced in the eighties a general Galois-cohomological obstruction to the existence of 0-cycles of degree 1 on \( X \), called the elementary obstruction. It vanishes if and only if the exact sequence of abelian groups \( 0 \to \bar{k}^* \to \bar{k}(X)^* \to \bar{k}(X)^*/\bar{k}^* \to 0 \) admits a Galois-equivariant splitting. In this talk we shall explain how this extension of Galois modules is related to period-index questions for principal homogeneous spaces under semi-abelian varieties. Applications will be given; in particular, when \( k \) is a number field, if \( X \) has points in every completion of \( k \) and the Tate-Shafarevich group of the Picard variety of \( X \) is finite, the elementary obstruction is then equivalent to the Brauer-Manin obstruction associated to locally constant algebraic classes of \( \text{Br}(X) \).