WILL IT EVER STOP HURTING?

\[
\frac{d\text{Pain}}{dt} = (-k_1 \text{Pain} + \frac{1}{e^{(e-k_2)/d}})(1 + \frac{1}{e^{(e-k_2)/d}})
\]

\[k_1 = ? \quad k_2 = ? \quad \hat{\sigma} = \text{How much she's still in my life}\]

PLEASE LET j ONLY BE A FEW DAYS ...

... OR WEEKS.

I GUESS THERE'S SOME KIND OF A
CUTOFF AFTER YEARS, WHERE IT STOPS
MATTERING AND WE CAN BE FRIENDS.
DO I WANT THAT?

IS \( k_1 \) POSITIVE? IS \( k_2 \) LARGE?

WILL I EVER STOP FEELING LIKE THIS?

(from xkcd)
1. Match the differential equations with the solution graphs labeled I-IV. You do not need to give reasons for your choices.

(a) \( y' = 1 + x^2 + y^2 \)  \hspace{1cm}  (b) \( y' = xe^{-x-y} \)

(c) \( y' = \frac{1}{1 + e^{x+y}} \)  \hspace{1cm}  (d) \( y' = \sin(xy) \cos(xy) \)

\[
\begin{array}{c}
\text{I} \\
\begin{array}{c}
\text{II} \\
\begin{array}{c}
\text{III} \\
\begin{array}{c}
\text{IV}
\end{array}
\end{array}
\end{array}
\end{array}
\]

(a) \( y'(0,0) = 1 \), slope is always positive and gets bigger as \( x \) and \( y \) do. III

(b) \( y'(0,0) = 0 \), but \( y = 0 \) is not a solution. I

(c) \( y'(0,0) = 1/2 \), slope is always positive and goes to zero as \( x \) and \( y \) get bigger. IV

(d) \( y'(0,0) = 0 \), and \( y = 0 \) is a solution. II

2. Solve the differential equation \( xy^2 y' = x + 1 \).

\[
\begin{align*}
y^2 dy &= \frac{x + 1}{x} dx \\
\int y^2 dy &= \int \left(1 + 1/x\right) dx \\
\frac{1}{3} y^3 &= x + \ln |x| + C
\end{align*}
\]

\[
y = \sqrt[3]{3x + 3 \ln |x| + 3C} = \sqrt[3]{3x + 3 \ln |x| + K}
\]
3. Consider the initial value problem \( y' = x(2 - y), y(0) = 1 \).

(a) Choose the correct direction field and use it to sketch the curve.

Any point with \( x = 0 \) has slope \( y' = 0(2 - y) = 0 \). So the direction field to the right (I) must be the correct choice. To sketch the curve, start at the point \((0, 1)\) and use the nearby slopes in the direction field as a guideline.

(b) Use Euler's method with a step size of 1 to estimate \( y(1) \).

We start with the point \((x_0, y_0) = (0, 1)\). We have step size \( h = 1 \). The slope at \((0, 1)\) is \( y' = 0(2 - 1) = 0 \). So

\[
x_1 = x_0 + h = 0 + 1 = 1
\]

and

\[
y_1 = y_0 + hy'(x_0, y_0) = 1 + 1 \times 0 = 1.
\]

This tells us that \( y(1) \approx 1 \).