We know that
\[
\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = \sum_{n=0}^{\infty} x^n \quad \text{for all } |x| < 1
\]

Use it to write the following related functions as power series. Make sure to specify the interval of convergence!

1. \[
\frac{2}{1-x} = 2 + 2x + 2x^2 + 2x^3 + 2x^4 + \cdots
\]
   It is a geometric series with first term 2 and ratio \(x\), so it converges when \(|x| < 1\).

2. \[
\frac{1}{1+x} = \frac{1}{1-(-x)}
\]
   \[
   \frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + (-x)^3 + (-x)^4 + \cdots = 1 - x + x^2 - x^3 + x^4 + \cdots
\]
   It is a geometric series with first term 1 and ratio \(-x\), so it converges when \(|-x| < 1 \implies |x| < 1\).

3. \[
\frac{1}{1-x^2}
\]
   \[
   \frac{1}{1-x^2} = 1 + (x^2) + (x^2)^2 + (x^2)^3 + (x^2)^4 + \cdots = 1 + x^2 + x^4 + x^6 + x^8 + \cdots
\]
   This is a geometric series with first term 1 and ratio \(x^2\), so it converges when \(|x^2| < 1 \implies |x| < 1\).

4. \[
\frac{x}{1-x}
\]
   \[
   \frac{x}{1-x} = x \cdot 1 + x \cdot x + x \cdot x^2 + x \cdot x^3 + \cdots = x + x^2 + x^3 + x^4 + x^5 + \cdots
\]
   This is a geometric series with first term \(x\) and ratio \(x\), so it converges when \(|x| < 1\).
5. \[
\frac{1}{2 + x} = \frac{1}{2(1 + x/2)}
\]

\[
\frac{1}{2(1 + x/2)} = \frac{1}{2} \left( 1 + \left( \frac{x}{2} \right) + \left( \frac{x}{2} \right)^2 + \left( \frac{x}{2} \right)^3 + \cdots \right) = \frac{1}{2} \left( 1 + x/2 + x^2/4 + x^3/8 + x^4/16 + \cdots \right)
\]

\[
= 1/2 + x/4 + x^2/8 + x^3/16 + x^4/32 + \cdots
\]

This is a geometric series with first term 1/2 and ratio x/2, so it converges when \(|x/2| < 1 \implies |x| < 2.|