1. (See 11.11 Example 3) Using the methods of Section 11.10, we can find that
\[(1 + x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \cdots, \quad |x| < 1\]

The function
\[K(v) = m_0c^2\left[\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right]\]
represents the kinetic energy of an object with resting mass \(c_0\) moving with velocity \(v\), according to special relativity. Note that \(c\) is the speed of light.

(a) Use the Maclaurin Series given above for \((1 + x)^{-1/2}\) to find the Maclaurin Series for \(K(v)\). What is its radius of convergence?

(b) If \(v\) is much smaller than \(c\) (which is basically always the case), then all terms after the first are very small when compared with the first term. If we omit them, what’s left? Does this look familiar?
2. In Section 11.10, we found that for all $x$,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

(a) Define $i = \sqrt{-1}$. Fill out the table of powers of $i$:

<table>
<thead>
<tr>
<th>$n$</th>
<th>$i^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>$i$</td>
</tr>
<tr>
<td>2</td>
<td>$-1$</td>
</tr>
<tr>
<td>3</td>
<td>$-i$</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>$i$</td>
</tr>
</tbody>
</table>

(b) Find the Maclaurin Series for $e^{ix}$.

(c) Find the Maclaurin Series for $\cos(x) + i\sin(x)$.
3. (See 11.10 Example 11) In Section 11.10, we found that for all $x$,

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$$

(a) Find the Maclaurin Series for $e^{-x^2}$.

(b) Integrate it to find the Maclaurin Series for

$$F(x) = \int_0^x e^{-t^2} \, dt.$$  

Note: There is no elementary way to write down the function $F(x)$. 