Fill in the blanks.

(1) If the series is of the form $\sum 1/n^p$, it is a $p$-series, which we know converges if $p > 1$ and diverges if $p \leq 1$.

(2) If the series has the form $\sum ar^{n-1}$ or $\sum ar^n$ it is a geometric series, which we know converges if $|r| < 1$ and diverges if $|r| \geq 1$.

(3) If the series has a form that is similar to (1) or (2), then one of the comparison tests should be considered. In particular, if $a_n$ is a rational function or an algebraic function of $n$ (involving roots of polynomials), the series should be compared with a $p$-series. Here’s the statement of the Comparison and Limit Comparison Theorems:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

(a) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all $n$, then $\sum a_n$ is also convergent.

(b) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all $n$, then $\sum a_n$ is also divergent.

(c) If $\lim_{n \to \infty} \frac{a_n}{b_n} = c$ where $c$ is a finite positive number, and $\sum b_n$ is convergent, then $\sum a_n$ is also convergent.

(d) If $\lim_{n \to \infty} \frac{a_n}{b_n} = c$ where $c$ is a finite positive number, and $\sum b_n$ is divergent, then $\sum a_n$ is also divergent.

(4) If you can see at a glance that $\lim_{n \to \infty} a_n \neq 0$, then the Test for Divergence should be used.

(5) If $a_n = f(n)$, where $\int_1^\infty f(x) \, dx$ is easily evaluated, then the Integral Test is effective (assuming the hypotheses of this test are satisfied). This tests says the following:

Suppose $f$ is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$.

(a) If $\int_1^\infty f(x) \, dx$ is convergent, then $\sum a_n$ is also convergent.

(b) If $\int_1^\infty f(x) \, dx$ is divergent, then $\sum a_n$ is also divergent.