Math 107. Homework #10. Solutions.

11.12.

1. **Hypotheses:** $H_0 : p = 0.23$; $H_A : p < 0.23$.
2. **Level of significance:** $\alpha = 0.05$
3. **Criterion:**

Reject H_0 if $\dfrac{x - 0.23n}{\sqrt{n(0.23)(1-0.23)}} < -z_\alpha = -1.645$.
Otherwise reserve judgment.

4. **Calculations:** We have $x = 72$, $n = 400$, and

$$\frac{72 - 0.23 \cdot 400}{\sqrt{400(0.23)(1-0.23)}} \sim -2.38 < -1.645.$$

5. **Decision:** Reject the null hypothesis.

11.16

1. **Hypotheses:** $H_0 : p_1 = p_2$; $H_A : p_1 \neq p_2$.
2. **Level of significance:** $\alpha = 0.05$
3. **Criterion:**

Reject H_0 if $\left|\dfrac{x_1/n_1 - x_2/n_2}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}}\right| > 1.96$.
Otherwise reserve judgment.

4. **Calculations:** We have $\hat{p} = \frac{141+123}{300+300} \sim 0.44$, and
$\left|\dfrac{141/300 - 123/300}{\sqrt{(0.44)(1-0.44)(1/300 + 1/300)}}\right| \sim 1.48 < 1.96.$
5. **Decision:** Reserve judgment.

11.22

1. **Hypotheses:** $H_0 : p_1 = p_2 = p_3 = p_4$; H_A : The p's are not all equal.
2. **Level of significance:** $\alpha = 0.05$
3. **Criterion:**

Reject H_0 if $\chi^2 \geq \chi^2_\alpha = 7.815$ ($4 - 1 = 3$ degrees of freedom). Otherwise reserve judgment.

4. **Calculations:** The table below lists the observed and the **expected** frequencies.

	A	B	C	D
complaining clients	49	54	41	48
	48	**48**	**48**	**48**
non-complaining clients	11	6	19	12
	12	**12**	**12**	**12**

Thus, $\chi_2 = \frac{(49-48)^2}{48} + \frac{(54-48)^2}{48} + \cdots + \frac{(12-12)^2}{12} \sim 8.96 > 7.815$.

5. **Decision:** Reject the null hypothesis.

Problem 4.

1. **Hypotheses:** H_0 : the height of a person does not influence his choice of a favorite airline; H_A: the height does influence the choice of an airline.

2. **Level of significance:** $\alpha = 0.05$

3. **Criterion:**

Reject H_0 if $\chi^2 \geq \chi^2_\alpha = 9.488$ ($(3-1)(3-1) = 4$ degrees of freedom). Otherwise reserve judgment.

4. **Calculations:** The table below lists the observed and the **expected** frequencies.

	Delta	United Airlines	Southwest
men shorter than 5'3"	16	17	11
	16.55	**16.55**	**10.9**
men between 5'3" and 6'1"	28	26	16
	26.35	**26.35**	**17.3**
men taller than 6'1"	20	21	15
	21.1	**21.1**	**13.8**

Thus, $\chi_2 = \frac{(16-16.55)^2}{16.55} + \frac{(17-16.55)^2}{16.55} + \cdots + \frac{(15-13.8)^2}{13.8} \sim 1.28 < 9.488$.

5. **Decision:** Reserve judgment.

Problem 5.

1. **Hypotheses:** H_0 : voters political preferences do not depend on the region he/she lives; H_A: voters political preferences are different for different regions.

2. **Level of significance:** $\alpha = 0.01$

3. **Criterion:**

Reject H_0 if $\chi^2 \geq \chi^2_\alpha = 13.277$ ($(3-1)(3-1) = 4$ degrees of freedom). Otherwise reserve judgment.

4. **Calculations:** The table below lists the observed and the **expected** frequencies.

	East North.	Central North.	West North.
HGSS	12387	15782	11383
	12340	**15818**	**11394**
VNPI	6173	7993	5783
	6225	**7977**	**5747**
LLLP	1100	1423	987
	1095	**1403**	**1012**

Thus, $\chi_2 = \frac{(12387-12340)^2}{12340} + \cdots + \frac{(987-1012)^2}{1012} \sim 1.88 < 13.277$.

5. **Decision:** Reserve judgment.