

Math 107. Homework #9. Solutions.

10.24.

1. **Hypotheses:** $H_0 : \mu = 4.2$; $H_A : \mu \neq 4.2$.

2. **Level of significance:** $\alpha = 0.05$

3. **Criterion:**

Reject H_0 if $\left| \frac{\bar{x} - 4.2}{s/\sqrt{n}} \right| > 2.262$, where 2.262 is the value of $t_{0.025}$ for $10 - 1 = 9$ degrees of freedom. Otherwise reserve judgment.

4. **Calculations:** From the sample we compute $\bar{x} = 4.00$, $s \sim 1.63$, and

$$\left| \frac{4.0 - 4.2}{1.63/\sqrt{10}} \right| \sim 0.39 < 2.262.$$

5. **Decision:** Not enough evidence to reject the null hypothesis.

10.26.

1. **Hypotheses:** $H_0 : \mu = 2.0$; $H_A : \mu < 2.0$.

2. **Level of significance:** $\alpha = 0.05$

3. **Criterion:**

Reject H_0 if $\frac{\bar{x} - 2.0}{s/\sqrt{n}} < -2.015$, where 2.015 is the value of $t_{0.05}$ for $6 - 1 = 5$ degrees of freedom. Otherwise reserve judgment.

4. **Calculations:** From the sample we compute $\bar{x} = 1.6$, $s \sim 0.43$, and

$$\frac{1.6 - 2.0}{0.43/\sqrt{6}} \sim -2.28 < -2.015.$$

5. **Decision:** Reject the null hypothesis.

10.28. (a) Two-sided alternative

1. **Hypotheses:** $H_0 : \mu = 4.4$; $H_A : \mu \neq 4.4$.

2. **Level of significance:** $\alpha = 0.05$

3. **Criterion:**

Reject H_0 if $\left| \frac{\bar{x} - 4.4}{s/\sqrt{n}} \right| > 2.093$, where 2.093 is the value of $t_{0.025}$ for $20 - 1 = 19$ degrees of freedom. Otherwise reserve judgment.

4. **Calculations:**

$$\frac{4.2 - 4.4}{0.5/\sqrt{20}} \sim -1.79, \quad \text{and} \quad 1.79 < 2.093.$$

5. **Decision:** Not enough evidence to reject H_0 .

(b) One-sided alternative

1. **Hypotheses:** $H_0 : \mu = 4.4$; $H_A : \mu < 4.4$.
2. **Level of significance:** $\alpha = 0.05$
3. **Criterion:**

Reject H_0 if $\frac{\bar{x} - 4.4}{s/\sqrt{n}} < -1.729$, where 1.729 is the value of $t_{0.05}$ for $20 - 1 = 19$ degrees of freedom. Otherwise reserve judgment.

4. **Calculations:**

$$\frac{4.2 - 4.4}{0.5/\sqrt{20}} \sim -1.79 < -1.729.$$

5. **Decision:** Reject H_0 .

(c) We should decide whether H_A should be one or two-sided **before** we see the data.

10.36. (a) Testing H_0 with $\alpha = 0.05$.

1. **Hypotheses:** $H_0 : \mu_1 = \mu_2$; $H_A : \mu_1 \neq \mu_2$.
2. **Level of significance:** $\alpha = 0.05$
3. **Criterion:** Reject H_0 if

$$z = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_1}}} \right| > 1.96,$$

otherwise reserve judgment.

4. **Calculations:**

$$z = \left| \frac{99 - 96}{\sqrt{\frac{(6.1)^2}{50} + \frac{(6.4)^2}{50}}} \right| \sim 2.40 > 1.96.$$

5. **Decision:** Reject H_0 . The difference between \bar{x}_1 and \bar{x}_2 is significant.

(b) Since $z = 2.40$ and we test a two-sided alternative hypothesis, the p -value for this problem is $2 \cdot (0.5000 - 0.4918) = 0.0164$. Since $0.0164 < 0.05$, we shall reject H_0 at the level of significance $\alpha = 0.05$.

10.42.

1. **Hypotheses:** $H_0 : \mu_1 = \mu_2; H_A : \mu_1 \neq \mu_2$.
2. **Level of significance:** $\alpha = 0.01$
3. **Criterion:** Reject H_0 if

$$z = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right| > 2.575,$$

otherwise reserve judgment.

4. **Calculations:**

$$\left| \frac{9.6 - 8.3}{\sqrt{\frac{(1.75)^2}{35} + \frac{(2.18)^2}{51}}} \right| \sim 3.06 > 2.575.$$

5. **Decision:** Reject H_0 . The difference between \bar{x}_1 and \bar{x}_2 is significant.

10.46.

1. **Hypotheses:** $H_0 : \mu_1 = \mu_2; H_A : \mu_1 \geq \mu_2$.
2. **Level of significance:** $\alpha = 0.05$
3. **Criterion:** Reject H_0 if

$$\frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} > 1.796,$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}},$$

and 1.796 is the value of $t_{0.05}$ for $6+7-2 = 11$ degrees of freedom. Otherwise reserve judgment.

4. **Calculations:** Since $s_p = \sqrt{\frac{5 \cdot 2^2 + 6 \cdot 4^2}{11}} \sim 3.245$ we have

$$\frac{14 - 12}{3.245 \sqrt{\frac{1}{6} + \frac{1}{7}}} \sim 1.107 < 1.796.$$

5. **Decision:** Not enough evidence to reject H_0 .