

Math 107. Fall 2006

Exam 1, September 28, 2006.

Problems and answers.

1. (2 points) Let $P(A) = 0.67$, $P(B) = 0.53$ and $P(A \cap B) = 0.33$. Draw a Venn diagram, fill in probabilities associated with various regions, and determine $P(A' \cup B)$ and $P(A' \cap B')$.

Answer: $P(A' \cup B) = 0.66$; $P(A' \cap B') = 0.13$.

2. (2 points) A student committee must consist of four juniors and three seniors.

a) If seven juniors and eight seniors are willing to serve on the committee, in how many different ways can it be selected?

Answer: ${}^7C_4 \cdot {}^8C_3 = \binom{7}{4} \binom{8}{3} = 1960$.

b) **Once the seven members of the committee have been selected** in how many ways they can choose among themselves the president and the vice-president?

Answer: $7 \cdot 6 = 42$.

3. (2 points) Suppose that we know that the average day temperature in July in Northumbria has the mean 89.3°F and its standard deviation is 5.6°F . What does Chebyshev's theorem tells us on the fraction of days in July in Northumbria during which the average day temperature is contained between 78.1°F and 100.5°F ?

Answer: 75%

4. (3 points) Five cards are drawn from an ordinary deck of 52 cards.

a) Find the probability that the hand contains precisely one diamond and two clubs.

Answer: $\frac{\binom{13}{1}\binom{13}{2}\binom{26}{3}}{\binom{52}{5}} \sim 0.1268.$

b) Find the probability that it contains at most one king.

Answer: $\frac{\binom{48}{5} + \binom{4}{1}\binom{48}{4}}{\binom{52}{5}} \sim 0.958.$

c) Find the probability that it contains precisely two hearts and no queens.

Answer: $\frac{\binom{12}{2}\binom{36}{3}}{\binom{52}{5}} \sim 0.192.$

5. (1 points) A box contains 3 red balls, 2 white balls, and 5 black ones. We select two balls from the box, one by one, **without replacement**. Find the probability that the first one is black and the second one is white.

Answer: $\frac{5}{10} \cdot \frac{2}{9} = \frac{1}{9}.$

6. (3 points) In an experiment, persons are asked to draw at random a ball from a hat containing ten balls numbered from 1 to 10, i.e., the sample space Ω can be written as $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $F = \{1, 2, 3, 5, 6\}$ and $G = \{1, 5, 7, 8\}$.

a) List the outcomes which comprise the event $F' \cup G'$. What is the probability of this event?

Answer: $F' \cup G' = \{2, 3, 4, 6, 7, 8, 9, 10\}; \quad P(F' \cup G') = 0.8.$

b) Are the events F and G independent?

Answer: Since $P(F \cap G) = 0.2 = 0.5 \cdot 0.4 = P(F) \cdot P(G)$, the events F and G are independent.

7. (1 point) For the last four consecutive days John has received 2, 7, 1, and 4, spam e-mail messages respectively. Find the median, the mean,

and the population standard deviation σ , of the number of spam messages John has got in these four days.

Answer: $\mu = 3.5$, $median = 3$, $\sigma = \sqrt{5.25} \sim 2.29$.

8. (3 points) In the following table employees of a company are classified according to their year income:

	< 30000\$	30000\$ – 80000\$	> 80000\$
Managers	1	3	10
Research Staff	10	11	5
Others	20	35	5

What is the probability that a randomly chosen employee:

a) earns more than 80000\$ a year?

Answer: $\frac{1}{5}$.

b) is a manager given she/he earns more than 80000\$ a year?

Answer: $\frac{1}{2}$.

c) earns less than 30000\$ a year given he/she is not a manager?

Answer: $\frac{15}{43}$.

9. (3 points) In a town there are three medicine doctors A, B, and C. For the last month they have written 34, 43, and 23 prescriptions, respectively, which have been filled in the only pharmacy store in that town. It is known that, because of clumsy handwriting, 30% of prescription written by doctor A are unreadable, and for doctors B and C the percentage of unreadable prescriptions are 10% and 20% respectively. Suppose that in the last month a pharmacist in the store got a prescription she could not read. What is the probability that it came from doctor C?

Answer: $\frac{0.2 \cdot 0.23}{0.3 \cdot 0.34 + 0.1 \cdot 0.43 + 0.2 \cdot 0.23} \sim 0.241$.