Math 107. Fall 2006
Exam 2, October 31, 2006.
Problems and answers.

1. (*2 points*) In the table below we list the probabilities that a certain kind of dog will have precisely n offsprings in one litter.

the number of offsprings n	0	1	2	3
the probability $f(n)$	0.26	0.20	0.32	0.22

Find the mean and the standard deviation for the number of offsprings in one litter.

Answer: $\mu = 1.5, \quad \sigma = 1.1.$

2. (*3 points*) Among a person's ten pairs of socks, four pairs need mending. If he randomly picks three pairs of sock to take along on a trip, what are the probabilities that:

(a) one pair of the socks will need mending?

Answer: *Since we deal with a hypergeometric distribution we have*

$$f(1) = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = 0.5.$$

(b) at least one pair will need mending?

Answer:

$$1 - f(0) = 1 - \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = \frac{5}{6} \sim 0.833.$$

(c) fewer than three pairs will need mending?

Answer:

$$1 - f(3) = 1 - \frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}} = \frac{29}{30} \sim 0.967.$$

3. (*2 points*) A psychologist wants to choose 5 persons for a team on which she will conduct some psychological experiment. Suppose that these five persons are selected at random from 478 students of a certain college, of whom 156 are men and 322 are women.

(a) What is the probability that the team will consist of two men and three women?

Answer:

$$\frac{\binom{156}{2}\binom{322}{3}}{\binom{478}{2}}.$$

(b) Approximate the above probability (that the team will consist of two men and three women) using the binomial approximation.

Answer: *We approximate the above hypergeometric distribution (with parameters $n = 5$, $a = 156$, $b = 322$) by the binomial distribution with $n = 5$, $p = \frac{a}{a+b} = \frac{156}{478} \sim 0.326$. Thus, the probability we are looking for are given by*

$$f(2) = \binom{5}{2}(0.326)^2(1 - 0.326)^{5-2} \sim 0.326.$$

4. (*4 points*) If the number of blossoms on a rare cactus is a random variable having the Poisson distribution with $\lambda = 1.6$, what are the probabilities that such a cactus will have:

(a) no blossoms?

Answer: $f(0) = e^{-1.6} \sim 0.202$.

(b) at least two blossoms?

Answer: $1 - f(0) - f(1) \sim 1 - 0.202 - 0.323 = 0.475$.

(c) more than one but less than four blossoms?

Answer: $f(2) + f(3) \sim 0.258 + 0.138 = 0.396$.

5. (*4 points*) The number of customers to whom a restaurant serves breakfast on a weekday morning is a random variable with $\mu = 142$ and $\sigma = 12$.

(a) According to Chebyshev's theorem, with what probability can we assert that between 112 and 172 customers will have breakfast there on a weekday morning?

Answer: *The probability is at least* $1 - \frac{1}{(2.5)^2} = 0.84$.

(b) Assume that the number of customers can be well approximated by the normal distribution with $\mu = 142$ and $\sigma = 12$. Use this fact to estimate the probability that the restaurant will serve a breakfast on a weekday morning to at least 126 and at most 145 customers.

Answer: *You have to use the continuity correction here. Since* $\frac{125.5-142}{12} = -1.38$ *and* $\frac{145.5-142}{12} = .29$*, the probability we are looking for is equal to* $0.4162 + 0.1141 = 0.5303$.

(c) Similarly as in (b), use the normal approximation to estimate the probability that the restaurant will serve a breakfast on a weekday morning to at most 138 customers.

Answer: *Again, we should use the continuity correction. Then we have* $\frac{138.5-142}{12} = -0.29$ *and for the probability we get* $0.5 - 0.1141 = 0.3859$.

6. (*2 points*) If the probability is 0.22 that a set of tennis will go into a tie breaker, what is the probability that at most two of five sets will go into tie breakers?

Answer: *We deal with the binomial distribution so*
$f(0) = \binom{5}{0}(0.22)^0(1-0.22)^5 \sim 0.2887$,
$f(1) = \binom{5}{1}(0.22)^1(1-0.22)^4 \sim 0.4072$,
$f(2) = \binom{5}{2}(0.22)^2(1-0.22)^3 \sim 0.2297$,
and for the probability that at most two tie breakers occur we get $f(0) + f(1) + f(2) \sim 0.9256$.

7. (*3 points*) Assume that the number of miles a driver gets on a set of radial tires is normally distributed with a mean of 30,000 miles and a standard deviation of 5000 miles. What is the probability that a driver using such a set of tires will get:

(a) at least 33,000 miles?

Answer: *You should* **not** *use the continuity correction here! Thus, since* $\frac{33000-30000}{5000} = 0.6$, *the answer is* $0.5000 - 0.2257 = 0.2743$.

(b) less than 29,000 miles?

Answer: *Again, no continuity correction is needed. Hence, since* $\frac{29000-30000}{5000} = -0.2$, *for the probability we get* $0.5000 - 0.0793 = 0.4207$.