

Math 107. Fall 2006
Exam 3, December 7, 2006.
Problems and answers.

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1. (*2 points*) A fisheries researcher wants to determine the mean weight of adult walleye in certain lake. He assumes that the standard deviation of the weight of walleyes is 1.5 lbs. How many walleyes does he have to examine to be sure that with probability at least 0.95 the error of his estimate is smaller than 0.5 lbs?

Answer:

$$n = \left(\frac{z_{\alpha/2}\sigma}{E} \right)^2 = \left(\frac{1.96 \cdot 1.5}{0.5} \right)^2 \sim 34.6.$$

We have to round it up so the answer is 35.

2. Nine bearings made by a certain process have the average diameter of 1.005 cm and the standard deviation of 0.004 cm.

(a) (1 point) Construct a 90% confidence interval for the mean diameter of bearing made by the given process.

Answer: *This is a small sample so we have to use $t_{\alpha/2} = 1.86$ (8 degrees of freedom). Since*

$$t_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 1.86 \cdot \frac{0.004}{\sqrt{9}} \sim 0.0025,$$

so we have $1.0025 < \mu < 1.0075$.

(b) (1 point) What can be said with 95% confidence about the maximum error if $\bar{x} = 1.005$ cm is used as an estimate of the mean diameter of bearings made by the given process?

Answer: *Since, again, this is a small sample, we use $t_{\alpha/2} = 2.306$ (8 degrees of freedom). Then*

$$E = t_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 2.306 \cdot \frac{0.004}{\sqrt{9}} \sim 0.0031.$$

3. (*3 points*) A horticulturist wishes to determine whether the honeybees visiting a peach orchard this year differ from the typical weight of 0.85 gram. A sample of 100 honeybees has mean weight of 0.90 gram with the sample standard deviation of 0.22 gram. At the level of significance 0.05, can he conclude that this year's bees differ from the typical weight?

Solution:

1. **Hypotheses:** $H_0 : \mu = 0.85$; $H_A : \mu \neq 0.85$.
2. **Level of significance:** $\alpha = 0.05$.
3. **Criterion:** *Reject H_0 if*

$$\left| \frac{\bar{x} - 0.85}{s/\sqrt{n}} \right| > z_{\alpha/2} = 1.96,$$

otherwise reserve judgment.

4. **Calculations:**

$$\frac{0.9 - 0.85}{0.22/\sqrt{100}} \sim 2.27 > 1.96.$$

5. **Decision:** *Reject the null hypothesis.*

4. (3 points) A brokerage company has two finalists for one trader position. The company likes candidate A better than candidate B, but will hire B if there is evidence that B is a more profitable trader. Looking at a random sample of 100 trades for each candidate the company found out that for A the mean profit was 5000\$ with the standard deviation 500\$, while for B the mean profit was 6000\$ with the standard deviation 5000\$. Determine if these samples show that the mean profit of B is significantly larger than A. Use $\alpha = 0.05$.

Solution:

1. **Hypotheses:** $H_0 : \mu_A = \mu_B$; $H_A : \mu_A < \mu_B$.
2. **Level of significance:** $\alpha = 0.05$.
3. **Criterion:** *Reject H_0 if*

$$\frac{\bar{x}_A - \bar{x}_B}{\sqrt{\sigma_A^2/n_B + \sigma_B^2/n_B}} < -z_\alpha = -1.645,$$

otherwise reserve judgment.

4. **Calculations:**

$$\frac{5000 - 6000}{\sqrt{(500)^2/100 + (5000)^2/100}} \sim -1.99 < -1.645,$$

5. **Decision:** *Reject the null hypothesis.*

5. (3 points) To test the effect of a physical fitness course on one's physical ability, the number of sit-ups that a person could do in one minute, both before and after the course, was recorded. Eight randomly selected participants scored as shown below. Can we conclude at the 0.01 level of significance that the physical ability was improved during the course?

Before	29	22	25	29	26	24	31	46
After	30	26	25	35	33	36	32	49

Solution: We need to use the test for paired data. Thus, first we compute the differences between the number of sit-ups before and after the course getting a sample of eight numbers: 1, 4, 0, 6, 7, 12, 1, 3. We have to check if this sample justify the assertion that $\mu > 0$.

1. **Hypotheses:** $H_0 : \mu = 0$; $H_A : \mu > 0$.
2. **Level of significance:** $\alpha = 0.01$.
3. **Criterion:** Reject H_0 if

$$\frac{\bar{x}}{s/\sqrt{n}} > t_\alpha = 2.998, \quad (7 \text{ degrees of freedom}).$$

Otherwise reserve judgment.

4. **Calculations:** Computations give $\bar{x} = 4.25$, $s = 3.99$. Hence

$$\frac{4.25}{3.99/\sqrt{8}} \sim 3.01 > 2.998.$$

5. **Decision:** Reject the null hypothesis.

6. (*3 points*) In order to determine whether or not the popping rate of popcorn is the same for two brands of popcorn one hundred kernels from each brand were put into a popcorn popper for the same amount of time. For brand A 20 kernel failed to pop, while for B 25 kernels failed to pop. Is this evidence, at the level of significance $\alpha = 0.1$, that the proportion of unpopped kernels for these two brands are different?

Solution:

1. **Hypotheses:** $H_0 : p_A = p_B; H_A : p_A \neq p_B$.
2. **Level of significance:** $\alpha = 0.1$.
3. **Criterion:** *Reject H_0 if*

$$\left| \frac{x_A/n_A - x_B/n_B}{\sqrt{\hat{p}(1 - \hat{p})(1/n_A + 1/n_B)}} \right| > z_{\alpha/2} = 1.645,$$

where $\hat{p} = \frac{x_A + x_B}{n_A + n_B}$. *Otherwise reserve judgment.*

4. **Calculations:** $\hat{p} = \frac{20+25}{100+100} = 0.225$

$$\frac{5000 - 6000}{\sqrt{(500)^2/100 + (5000)^2/100}} \sim -0.846 > -1.645,$$

5. **Decision:** *Reserve judgment.*

7. (4 points) A psychologist would like to know if there is a relation between student's major and his/her "movie preferences". In order to answer this question she showed to 100 students, who graduated in Mathematics, Biology and English Literature respectively, three movies and asked them to decide which one, in their opinion, is the best. The results she has got are as follows

	Citizen Kane	Gone with the Wind	Lawrence of Arabia
Mathematics	15	5	10
Biology	12	3	15
English	18	8	14

Can she claim, at the level of significance $\alpha = 0.05$, that students' movie preferences depend on the major they chose?

Solution:

1. **Hypotheses:** H_0 : Students' movie preferences do not depend on their major;

H_A : Students' movie preferences do depend on their major;

2. **Level of significance:** $\alpha = 0.05$

3. **Criterion:**

Reject H_0 if $\chi^2 \geq \chi_\alpha^2 = 9.488$ ($(3 - 1)(3 - 1) = 4$ degrees of freedom). Otherwise reserve judgment.

4. **Calculations:** The table below lists the observed and the expected frequencies.

	<i>Citizen Kane</i>	<i>Gone with the Wind</i>	<i>Lawrence of Arabia</i>
<i>Mathematics</i>	15 13.5	5 4.8	10 11.7
<i>Biology</i>	12 13.5	3 4.8	15 11.7
<i>English</i>	18 18	8 6.4	14 15.6

Thus, $\chi_2 = \frac{(15-13.5)^2}{13.5} + \frac{(5-4.8)^2}{4.8} + \dots + \frac{(14-15.6)^2}{15.6} \sim 2.76 < 9.488$.

5. **Decision:** Reserve judgment.