

Math 107. Review for the first midterm
Solutions to the problems which are not solved in the book.

R.2 (a) At least $1 - \frac{1}{(1.2)^2} = 0.3056$.

(b) At least $1 - \frac{1}{(6.25)^2} = 0.9744$.

(c) At least $1 - \frac{1}{11^2} = 0.9917$.

R.13 (a) $\bar{x} = \frac{0.51}{8} = 0.06375$, $s = \sqrt{0.00001907} \sim 0.0044$,

(b) The median is 0.0625, the range is 0.012.

R.19 By Chebyshev's inequality, the probability is at least $1 - \frac{1}{4^2} = \frac{15}{16}$.

R.54 $\frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{1}{30}$.

R.55 (a) $4 \cdot 3 \cdot 4 = 48$.

(b) $3 \cdot 4 = 12$.

(c) $4 \cdot 3 \cdot 2 = 24$.

R.58 $0.20 + 0.25 - 0.15 = 0.30$.

R.59 $7 \cdot 6 \cdot 5 = 210$ ways.

R.62 (a) $P(A) = \frac{180}{380} = \frac{9}{19} \sim 0.474$.

(b) $P(R') = \frac{310}{380} = \frac{31}{38} \sim 0.816$.

(c) $P(A \cap R) = \frac{20}{380} = \frac{1}{19} \sim 0.053$.

(d) $P(A|R) = \frac{20/380}{70/380} = \frac{2}{7} \sim 0.286$.

(e) $P(A' \cup R) = \frac{220}{380} = \frac{11}{19} \sim 0.579$.

(f) $P(R'|A) = \frac{160/380}{180/380} = \frac{8}{9} \sim 0.889$.

R.69 (a) $\frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{256} \sim 0.039$.

(b) $\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} = \frac{17160}{6497400} \sim 0.0026$.

R.71 (a) $\binom{6}{4} = 15$ ways.

(b) $\binom{6}{2} \cdot \binom{6}{2} = 225$ ways.

(c) $\binom{12}{4} = 495$ ways.

Problem.

We select at random 5 cards from a deck of 52 cards. Find the probability that:

(i) we will have at most one ace;

Answer: $\frac{\binom{48}{5} + \binom{4}{1}\binom{48}{4}}{\binom{52}{5}}$.

(ii) we will have precisely one spade and two clubs;

Answer: $\frac{\binom{13}{1}\binom{13}{2}\binom{26}{2}}{\binom{52}{5}}$.

(iii) we will have three diamonds and suits of the two remaining cards are different.

Answer: $\frac{\binom{13}{3}\binom{3}{2}\binom{13}{1}\binom{13}{1}}{\binom{52}{5}}$.