

The Power of Omaha and Florida in the 2008 U.S. Presidential Election

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1 Introduction

The United States, uniquely among all countries, elects its head of state using a two-tiered system called the Electoral College. If we view the states that make up the Electoral College as voters and make some simplifying assumptions, it is possible to use the theory of voting and voting power to measure the power of the individual states in a presidential election. Modeling the Electoral College in this way and measuring the power of the states has been done by several authors, including J. Banzhaf [2] and I. Mann and L. Shapley [7]. However, this type of model uses only the formal rules of an election and does not take into account the particulars of a given election, for example, how likely a given state is to vote for a particular candidate.

One of the simplifying assumptions that is made in this type of modeling is that every state uses a winner-take-all system to award its electoral votes. In theory, two states—Maine and Nebraska—do not use a winner-take-all system, although until recently their votes had never been split. However, in the 2008 presidential election, Nebraska split its electoral votes: Barack Obama received one electoral vote from the second congressional district (the district containing Omaha), while John McCain received the remaining electoral votes from Nebraska. One implication of this is that models of voting power in the Electoral College up to this point do not accurately apply to the 2008 election, since they assume no splitting of electoral votes.

In this paper we discuss yes-no voting systems and the Banzhaf power index, and look at how these have been applied to the Electoral College. Then we look at a way to use these ideas to model power in the 2008 U.S. presidential election, by taking into account the probability that a given voter—in this case a state or congressional district—votes for one of the two major candidates. Because of the

particulars of the 2008 election, we can apply our model allowing for the splitting of electoral votes in Maine and Nebraska. It turns out that in our model, Omaha and the smallest states have a higher percentage of the power relative to their weight in the electoral college, while Florida has a substantially smaller percentage of the power than its weight.

2 Yes-No Voting Systems

A yes-no voting system is a mathematical model of voting by legislative and other decision-making bodies in which voters vote “yes” or “no” on single alternatives, such as bills or amendments. Many legislative systems in real life can be modeled as yes-no voting systems. An election with only two candidates can be modeled as a yes-no voting system by viewing the election as deciding “yes” or “no” on one candidate. Details of yes-no voting systems can be found in [9, Chap. 3] (from a mathematical point of view) and in [3] (from a political science point of view). Note that in [3] the authors use the term “simple voting game” for a yes-no voting system.

Informally, a yes-no voting system is a set of rules that specifies exactly which sets of “yes” votes allow for passage of the bill or motion in front of the voting body. The standard way to define such a system is to specify the **winning coalitions**, the sets of voters that can assure passage if all vote yes.

Definition 1. A **yes-no voting system** consists of a finite set of **voters** V along with a collection \mathcal{W} of subsets of V , the **winning coalitions**, with the following properties:

1. $V \in \mathcal{W}$ and $\emptyset \notin \mathcal{W}$
2. **Monotonicity.** If $X \subseteq Y$ and $X \in \mathcal{W}$, then $Y \in \mathcal{W}$.

A **coalition** is any subset $U \subseteq V$. If $U \notin \mathcal{W}$ then U is a **losing coalition**. If $U \in \mathcal{W}$, a voter $v \in U$ is **critical in** U if $U \setminus \{v\}$ is losing. A coalition U is a **minimal winning coalition** if $U \in \mathcal{W}$ and every voter in U is critical.

Remarks 1. 1. We can view a yes-no voting system as a set of rules for passage of a motion. Each voter votes “yes” or “no” and the motion passes iff the set of voters who voted “yes” is in \mathcal{W} . Property 1 says that if everyone votes “yes”, then passage is assured and no motion can pass without at least one “yes” vote.

2. Monotonicity simply says that if there are enough “yes” votes for passage, and more voters vote “yes”, then the motion still passes. Note that by monotonicity a coalition is winning iff it contains a minimal winning coalitions and thus we need only specify the minimal winning coalitions when defining a yes-no voting system.

Example 1. The United Nations Security Council can be viewed as a yes-no voting system with 15 voters, the 5 permanent members and 10 nonpermanent members.

Passage of a motion requires at least 9 “yes” votes with each permanent member having veto power. Thus the minimal winning coalitions consist of the 5 permanent members plus any 4 nonpermanent members.

Example 2. The U.S. Federal system is a yes-no voting system with voters the 100 senators, 435 members of the House of Representatives, the President, and the Vice President. Passage of a bill requires a majority vote in both the senate and the house, with the Vice President casting the tie-breaking vote in the senate. The President can veto a bill, however the veto can be overridden with a two-thirds vote in both the house and the senate. Hence there are three types of minimal winning coalitions: 51 senators, 218 representatives and the President; 50 senators, 218 representatives, the Vice President, and the President; 67 senators and 290 representatives.

2.1 Weighted Voting Systems

In many voting systems, the preferences of some voters might carry more weight than others. For example, in the U.S. Electoral College, the number of electoral votes that a state controls is related to the population of the state. Weighted voting systems are a model of yes-no voting systems in which voters have varying numbers of votes and there is a threshold number of votes that must be reached for passage of a motion.

A yes-no voting system (V, \mathcal{W}) is a **weighted voting system** if we can assign to each voter $v \in V$ a **weight** $w_v \in \mathbb{R}^+$, and find $q \in \mathbb{R}^+$, the **quota**, such that a coalition U is in \mathcal{W} iff $\sum_{v \in U} w_v \geq q$.

In many cases, yes-no voting systems are given explicitly as weighted voting systems, while for others in order to prove that they are weighted we must find weights and a quota which work. It is easy to see that the U.N. Security Council (Example 1) is a weighted voting system: Let x denote the weight of a permanent member, y the weight of a nonpermanent member, and q the quota, then any solution to the system of inequalities

$$4x + 10y < q, \quad 5x + 4y \geq q$$

will do the job. For example $x = 7$, $y = 1$, and $q = 39$ works.

Suppose we have two winning coalitions in a weighted voting system and we make a one-for-one swap between the coalitions, i.e., we swap a voter from one coalition (not in the other) for a voter in the other coalition (not in the first). Then at least one of the two new coalitions must be winning. This is because the sum of the total weights of the coalitions cannot change and since this sum is at least $2q$, the total weight of at least one coalition must be at least q after the swap.

A yes-no voting system is **swap robust** if the property we just described holds: Given two winning coalitions, making a one-for-one swap of voters between the coalitions leaves at least one coalition winning. The remarks in the previous paragraph show that any weighted voting system is swap robust.

We can now show that the U.S. Federal system (Example 2) is not a weighted voting system because it is easy to see that it is not swap robust. For example, if one coalition consists of the 67 tallest senators and 290 tallest representatives, while another coalition consists of the 67 shortest senators and the 290 shortest representatives, then both coalitions are winning. But swapping the tallest senator in the first coalition for the shortest representative in the second leaves both coalitions losing. This example was taken from [9, §2.4].

3 Banzhaf power in yes-no voting systems

In 1965, the Board of Supervisors of Nassau County, New York used a weighted voting system to pass laws and bills. The voters were the six districts of the county with weights proportional to the population of the districts. Majority rule was used for passage, so that the quota was half of the total weights plus one. The districts and weights are listed in Table 1.

| District | Weight |
|-----------------|--------|
| Hempstead 1 | 31 |
| Hempstead 2 | 31 |
| Oyster Bay | 28 |
| North Hempstead | 21 |
| Long Beach | 2 |
| Glen Cove | 2 |

Table 1: Nassau County Board of Supervisors, 1965

With a total of 115 votes, this makes the quota 58, hence the minimal winning coalitions consist of any two of the three larger districts. In other words, the three smaller districts will never be critical in any coalition. Practically speaking, this means that the votes of the three smaller districts never matter for the passage of a motion, and that the votes of the three larger districts are equally important.

In 1965, John Banzhaf, a lawyer and professor of law specializing in public interest law, filed a lawsuit against the Nassau board, arguing that all of the power within the board was equally distributed among the three larger districts. Banzhaf proposed a method for measuring how power is distributed in yes-no voting systems [1]. His idea was that the power of a voter arises from the frequency with which the voter is necessary to keep a winning coalition from being losing. Eventually, after a series of lawsuits, New York State’s highest court made consideration of the Banzhaf index a legal requirement for approving voting reapportionment plans.¹ In fact, Banzhaf’s method for measuring power in yes-no voting systems first appeared in a little-noticed 1946 paper by L. Penrose [10]. For more on the history of measuring power in voting systems, see [3, §1.2].

¹Iannucci v. Board of Supervisors, 229 N.E. 2d 195 (1967)

Definition 2. Suppose (V, \mathcal{W}) is a yes-no voting system and $v \in V$. Recall that if $v \in U$ for U a winning coalition, then v is critical (in U) if $U \setminus \{v\}$ is losing.

1. The **Banzhaf power** of v , denoted $BP(v)$, is the number of winning coalitions in which the voter is critical.
2. The **total Banzhaf power** of the system, denoted by TBP , is $\sum_{v \in V} BP(v)$.
3. The **Banzhaf power index**, or **BPI**, of v is $\frac{BP(v)}{TBP}$.

Example 3. For the Nassau County Board of Supervisors in 1965, as noted above, any one of the three larger districts is critical in a winning coalition when that coalition contains the district plus exactly one other large district, and the smaller districts are never critical. This yields BPI of each of the three larger districts of $\frac{1}{3}$ with BPI of 0 for the smaller districts.

Other measures of power in yes-no voting systems have been defined. The most famous is the Shapley-Shubik power index [12], which uses the notion of a “pivotal” voter. Here one considers winning coalitions being formed one voter at a time, then the pivotal voter is the one that shifts the coalition from losing to winning. The Banzhaf and Shapley-Shubik power indices are the most famous and most widely used, but there are others. For more examples, see [9, Chap. 9] and [3, §6.4]. We focus on the Banzhaf index in this paper because it is considered by experts in voting theory to be more appropriate for evaluating power in the U.S. Electoral College than the equally famous Shapley-Shubik index.

4 The U.S. Electoral College

Presidential elections in the United States are decided by an institution called the Electoral College, which is very close to being a weighted voting system. Each state has a number of “electoral votes” equal to the number of members of Congress the state has (senators plus representatives) and, in addition, the District of Columbia (D.C.) has three electoral votes. This yields a total of 538 electoral votes, which are actually cast by individuals called “electors” who meet a few weeks after the November presidential election. The winner is decided by majority rule, so that a candidate becomes president if they receive 270 or more electoral votes. If no candidate receives 270 votes, the U.S. House of Representatives decides on the winner. This has happened twice, in 1800 and 1824.

With two exceptions, each state awards all of its electoral votes to the candidate who wins a plurality of the popular vote in that state. The exceptions are Maine and Nebraska, which award one electoral vote to the plurality winner in each congressional district — currently two for Maine and three for Nebraska — with the remaining two electoral votes going to the overall popular vote winner in the state.

If we assume that there are only two presidential candidates receiving electoral votes and that Maine and Nebraska do not split their electoral votes, the U.S. Electoral College can be viewed as a weighted yes-no voting system with voters the individual states and weights the number of electoral votes. This is a model that has been used to study U.S. presidential elections and power in the Electoral College.

Because of the size of the Electoral College, it is not possible, even using a computer, to compute Banzhaf power using the definition directly. Various methods for calculating exactly and approximately have been developed, including approximate methods using Monte Carlo computer simulations [7] and exact methods using generating functions [8]. We make use of computer algorithms developed by D. Leech [6], that use generating functions, are exact, and which are available online.

Using the current electoral votes for each state, the Banzhaf power index ranges from .0055 for the states with 3 electoral votes to .114 for California, which has the most electoral votes (55). We measured the difference between the percentage of the electoral votes ev that a state has and the percentage of the Banzhaf power bp it has by calculating the percent increase/decrease, i.e., the “% increase” in Table 2 denotes $\frac{bp - ev}{ev} \times 100$. The percentage of the power that each state has is very close to the percentage of the electoral votes that the state has, except for California, which has a significantly higher percentage of the Banzhaf power than electoral votes. Table 2 gives the percentages for some of the states.

| Electoral Votes | % of electoral votes | BP % | % increase |
|--------------------|----------------------|--------|------------|
| 55 (CA) | 10.22% | 11.40% | 11.55% |
| 34 (TX) | 6.32% | 6.39% | 1.1 % |
| 27 (FL) | 5.76% | 5.79% | 0.52% |
| 21 (PA) | 3.90% | 3.87% | -0.77% |
| 15 (NC, GA) | 2.79% | 2.74% | -1.79% |
| 3 (D.C., MT, etc.) | 0.56% | 0.55% | -1.79% |

Table 2: Banzhaf power in the Electoral College

As we saw from the Nassau County example in §3, in a weighted voting system the amount of power that a voter has is not necessarily directly related to the voter’s weight. However, the above data shows that Banzhaf power in the Electoral College is, apart from in California, very closely tied to the number of votes that the state has. In general, the larger states have a slightly higher percentage of the power than their percentage of the electoral votes, while the smaller states have a slightly lower percentage.

5 Power and Safe States

In this section we look at a way to calculate Banzhaf power in the Electoral College in the 2008 U.S. presidential election which takes into account the fact that some states were virtually certain to go to one or the other of the two major candidates. Instead of viewing each state as a voter, we look only at the so-called swing states, the states for which both candidates have at least some chance of winning. The new material in this section is taken from [11].

In the previous section we saw that California has the most Banzhaf power and is the only state for which the percentage of power is significantly higher than its percentage of the electoral votes. However, if we look at what happens in an actual presidential election, this does not seem to be reflected by factors such as the amount of time and money spent by the candidates in California, and in some of the other states with a large share of the Banzhaf power. This is, of course, due to the fact for some of the states it was already assumed that they would vote for one candidate or the other. California is considered a “safe” Democratic state, so given the winner-take-all system for awarding electoral votes, neither candidate saw any purpose in spending much time or money campaigning there.

Informally, Banzhaf power is measuring the extent to which an individual voter can control the outcome of a decision. The BPI of a voter can be thought of as the probability that the voter will cast the deciding “yes” vote assuming that all voters vote randomly. Political scientists refer to this as the *a priori* power in the voting system; it depends only on the formal rules of the system and does not take into account the probability that a particular voter will vote “yes” or “no” in a given situation.

The fact that measuring *a priori* power in the U.S. Electoral College is not a good model of actual elections has been noted by several authors, and various solutions have been proposed. For example, in [4], A. Gelman, J. Katz, and F. Tuerlinckx develop statistical models of voting in U.S. presidential elections. D. Strömberg [13] develops a two-candidate probabilistic model which assigns probabilities that a voter in a given state will vote for the Democratic or Republican candidate, using demographic information about individual states, their voting history, and other factors.

All of the models of power in a presidential election mentioned so far are models of a generic presidential election, which can then be applied to any particular election. Further, all of them assume that the electoral votes in Nebraska and Maine are not split, and thus they cannot be applied to the 2008 election. We propose a different approach, namely, to look at the 2008 presidential election specifically, and to measure the power of the “swing states” in deciding the election. The idea is simple: We define “safe states” for each of the two major candidates and assume that the electoral votes in the safe states have already been won by the candidates. We then view the remaining states—the “swing states”—as voters in a weighted voting system and calculate their Banzhaf power indices. Because of the particulars

| Voter | Electoral Votes | BP % | % of electoral votes | % increase |
|------------|-----------------|--------|----------------------|------------|
| NE#1, NE#2 | 1 | .49% | .46% | 6.52% |
| ND, SD | 3 | 1.56% | 1.47% | 6.12% |
| NH | 4 | 2.07% | 1.95% | 6.15% |
| NM, NV, WV | 5 | 2.44% | 2.30% | 6.09% |
| SC | 8 | 3.87% | 3.69% | 4.88% |
| CO, LA | 9 | 4.33% | 4.15% | 4.34% |
| AZ, MN, WI | 10 | 4.79% | 4.61% | 3.90% |
| IN, MO | 11 | 5.24% | 5.07% | 4.59% |
| VA | 13 | 6.11% | 5.99% | 2.00% |
| GA, NC | 15 | 6.93% | 6.91% | -.29% |
| OH | 20 | 8.72% | 9.22% | -5.42% |
| PA | 21 | 9.01% | 9.68% | -6.92% |
| FL | 27 | 10.27% | 12.44% | -17.44% |

Table 3: Banzhaf power of the swing states

of the 2008 election, we can apply our model allowing for the splitting of electoral votes in Maine and Nebraska.

We need an objective method for deciding which states are safe for Obama and which for McCain. We use probabilities calculated using a model developed by N. Silver, founder and owner of the website <http://www.fivethirtyeight.com>. This model had the most accurate predictions in the 2008 election and also calculated separate probabilities for the congressional districts of Maine and Nebraska as well as a probability for these states at-large. The model uses polling data, weighting newer polls more heavily, along with demographic information and voting history of the state. Our calculations were done after the election, and we used the final probabilities given, which were calculated a few days before the election. Since we wanted to insure that a safe state really was safe for the candidate, at least over the last month or so before the election, we assumed a state was safe if the calculated probability for a candidate was 100% and there was at most one poll between October 1 and the election giving that candidate a less than 10% margin.

The safe states for Obama were CA, CT, DC, DE, HI, IA, IL, MA, MD, ME, MI, NJ, NY, OR, RI, VT, and WA, which yields a total of 211 electoral votes. The safe states for McCain were AK, AL, AR, ID, KS, KY, MS, NE #3, NE, OK, TN, TX, UT, and WY, a total of 105 electoral votes. Using our criteria, both districts of Maine were safe for Obama, and the third district of Nebraska as well as Nebraska at-large ² were safe for McCain. Thus the electoral college becomes a weighted yes-no voting system with 23 voters with total weight 217. The weights of the voters are as follows: 1, 1, 3, 3, 3, 4, 5, 5, 5, 8, 9, 9, 10, 10, 10, 11, 11, 13, 15, 15, 20, 21,

²McCain's margin in NE #3 was large enough to overcome much closer margins in NE #1 and #2 and make NE overall safe for him.

27.

We (arbitrarily) view the election as a yes/no referendum on Obama as president and thus the quota is $270 - 211 = 59$ in this case. To calculate the power indices, we use the software at <http://www.warwick.ac.uk/~ecaae/>. The results are in Table 3; we give the percentage of Banzhaf power along with the percentage of the 217 electoral votes and the percent increase between power and electoral vote percentage, as we did in Table 2.

The results are quite different from the results for a generic election: In our model of the 2008 presidential race, the smaller swing states had an advantage over the larger swing states. In fact, the function is almost monotone decreasing. Surprisingly, Florida, the largest swing state, had a huge drop between its percentage of power and percentage of votes in the “swing state” Electoral College. Of course, Florida still has the highest percentage of the Banzhaf power, which means that in our model it was the most likely to be the deciding state in the election. But this indicates that Florida’s power to decide the election was less than it should have been, given the number of electoral votes it has.

Finally, we think that it would be interesting to calculate power in this way for recent elections such as 2000 and 2004 and compare to 2008. Unfortunately, the probabilities we used to find the swing states are not available for previous elections. We hope to explore this in a future project.

For further reading on mathematical voting theory and measuring power in voting systems, we highly recommend [5] and [9].

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