Warren Shull Teaching Statement

To understand a mathematical idea is to internalize the logical steps leading to it from our common sense. I consistently aim to guide students toward this understanding, especially when exploring big-picture patterns at the core of the content. As one student wrote to me in an email, “I can now confidently say that I understand the concepts behind Calc 1, and... I truly believe this one semester has completely changed the way I feel about math.” I strive to give students the necessary tools to continue learning on their own, making sense of any new mathematical ideas they may encounter and communicating what they know to others, both verbally and in writing.

Over my years of teaching, few lessons have become as clear as the importance of meeting students where they are. A math course, or a sequence of them, might be imagined as a stepladder for students to climb. While we cannot force them to ascend, we can make the process more fulfilling by the kind of ladder we build. In particular, it helps to make sure the bottom rung is within every student’s reach. To that end, I begin each semester trying to gauge students’ understanding of prerequisite material, using either a sample final exam from the previous course or a set of diagnostic exercises. When I deem it necessary, I have experimented with various ways to balance early class days between old and new content, often systematically grouping them by topic. For example, I often review exponents and introduce logarithms on the same day to emphasize how they are connected. Often, this ends with students thinking on their own or in small groups, working to derive logarithm properties from exponent properties.

Such interactive approaches allow me to observe students as they reason through a problem. This helps me understand their thought processes and offer appropriate guidance while allowing them to tackle as much of the problem as possible on their own. For instance, when I introduce curve sketching and concavity, I assign each student a basic qualitative graph shape, asking them to determine whether each of the first two derivatives is positive, negative, or zero. Students invoke what they know, from the course as well as their common sense, tracing the logical steps to their conclusion, then comparing their answers with a nearby classmate. After discussing each conclusion together as a class until it makes sense to the students, I invite them to look for a pattern among the shapes based on their second derivative. The descriptions students offer are generally on the right track, and only then do I introduce the terms “concave up” and “concave down.”

When preparing a lecture, I think carefully about the best ways to visualize each important concept, refining my visual displays over time. I also make good use of analogies to ideas likely to be more familiar. When introducing derivatives, for example, I draw a series of images of the same point on the same curve, “zooming in” on the point until we are looking at a straight line. The pictures’ size makes them memorable, and working on the board also allows for flexibility. It also forces a slower pace, helping to build students’ confidence. As a very relatable analogy, I then mention that the apparent flatness of the ground around us is because we are so far zoomed into Earth’s surface. When I then show an example of a nonexistent derivative, such as the absolute value function, the analogy extends into asking, “What if Earth were a cube (and you lived right at the edge)?”

A common kind of algebra mistake involves omission or misplacement of parentheses. When this happens, I always highlight the underlying meaning of what is written. (“Are you adding a to b and multiplying the result by c, or multiplying b by c and adding a to the result?”) To aid in student
understanding, I often draw rectangular boxes around expressions that must remain grouped together. Applied to functional notation, I find that this helps many students understand function composition and the Chain Rule. In particular, the “outside derivative” part of the Chain Rule can be carried out by simply treating an empty box as a variable, filling it back in at the end. Furthermore, for complicated expressions involving multiple uses of the Chain Rule, drawing different such boxes in different colors tends to be very illuminating for many students.

An important feature of mathematics, and an important benefit of mathematical thinking, is the ability to “see the common skeleton shared by problems that look completely different on the surface,” as Jordan Ellenberg once wrote. In other words, parts that look very different end up fitting together in the same arrangement. When students are tripped up by these surface-level differences, such as variable and function names, I always try to redirect their attention to the role a number or expression plays in a formula, and the overarching structure that ties them all together. Here, I find, using a different color for each role in the formula, and keeping the same color scheme from one problem to the next, makes students better able to see this common skeleton.

While I strive to give calculus students a robust intuition for limits and derivatives, their formal definitions are commonly omitted from the curriculum. As valuable as these intuitions are, students armed with the formal logic to back them up become even better thinkers. Fortunately, in applying the Mean Value Theorem to certain problems, calculus students see good examples of formal logic confirming their intuitions.

When teaching upper level courses, I hope to accomplish many similar goals along with some additional ones. In any proof course, introductory or otherwise, I intend to forge a strong connection between students’ intuition on each topic and the formal logic needed to build proofs out of definitions and earlier theorems. Beyond that, I will aim to give students enough practice for formal logic itself to become intuitive, to the point where it can bridge the gap from their intuition to even the least intuitive arguments and results.

In the coming decades, I foresee interactive approaches becoming more common. As lecture videos become more accessible and better, the best course instructors will be those who can artfully probe each student’s understanding and strategically nudge it along. I have done this effectively with a variety of learners, precisely because I am listening to each student’s thought process and adapting the timing and phrasing of my questions accordingly. This skill will be valuable as individual students continue to differ in background and in how their minds work, while course content changes more gradually and the underlying math remains true throughout time.