Hey Bill, what’s the deal with *The chromatic number of finite type-graphs*?

We say that sets $X, Y \in \binom{[n]}{k}$ have order type $\tau \in [3]^\ell$ if $|X \cup Y| = \ell$ and $\tau_i = 1, 2, \text{ or } 3$ whenever the $i$th element of $X \cup Y$ is in $X \setminus Y$, $Y \setminus X$, or $X \cap Y$ respectively. The type graph $G(n, \tau)$ is the graph with vertex set $\binom{[n]}{k}$ where $X$ is adjacent to $Y$ if and only if $X$ and $Y$ have type $\tau$. The chromatic number of type graphs have been studied extensively by Erdős, Hajnal, Rado, and others. More recently, Avart, Luczak, and Rödl asked if there was a general formula for the chromatic number based $\tau$. Their question was answered by the following:

**Theorem 1** (Avart, K., Reiher, Rödl). *For every type $\tau$, there exists a constant $\beta = \beta(\tau)$ so that:*

$$\chi(G(n, \tau)) = \Theta(\log_{(\beta)} n)$$

*where $\log_{(t)} n$ is the $t$-fold iterated binary logarithm of $n$."

Given $\tau$, we provide an algorithm to (easily) compute $\beta$. 