Stacks
Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure

- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

Example: ADT modeling a simple stock trading system

- The data stored are buy/sell orders
- The operations supported are:
  - order `buy`(stock, shares, price)
  - order `sell`(stock, shares, price)
  - void `cancel`(order)

- Error conditions:
  - Buy/sell a nonexistent stock
  - Cancel a nonexistent order
The Stack ADT

- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - `push(object)`: inserts an element
  - `pop()`: removes and returns the last inserted element
- Auxiliary stack operations:
  - `top()`: returns the last inserted element without removing it
  - `size()`: returns the number of elements stored
  - `isEmpty()`: indicates whether no elements are stored
Stack Interface in Java

- Java interface corresponding to our Stack ADT
- Requires the definition of class EmptyStackException
- Different from the built-in Java class java.util.Stack

```java
public interface Stack<E> {
    public int size();
    public boolean isEmpty();
    public E top() throws EmptyStackException;
    public void push(E element);
    public E pop() throws EmptyStackException;
}
```
Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception.
- Exceptions are said to be “thrown” by an operation that cannot be executed.
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty.
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException.
Applications of Stacks

- **Direct applications**
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine

- **Indirect applications**
  - Auxiliary data structure for algorithms
  - Component of other data structures
Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack.
- When a method is called, the JVM pushes on the stack a frame containing:
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack.
- Allows for recursion

```java
main() {
    int i = 5;
    foo(i);
}

foo(int j) {
    int k;
    k = j+1;
    bar(k);
}

bar(int m) {
    ...
}
```

© 2010 Goodrich, Tamassia
Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Algorithm \textit{size}()
\begin{verbatim}
return \( t + 1 \)
\end{verbatim}

Algorithm \textit{pop}()
\begin{verbatim}
if \textit{isEmpty}() then
    throw \textit{EmptyStackException}
else
    \( t \leftarrow t - 1 \)
    return \( S[t + 1] \)
\end{verbatim}
Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm $push(o)$

```
if $t = S.length - 1$ then
  throw FullStackException
else
  $t \leftarrow t + 1$
  $S[t] \leftarrow o$
```
Performance and Limitations

- **Performance**
  - Let $n$ be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$

- **Limitations**
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception
Array-based Stack in Java

```java
public class ArrayStack<E> implements Stack<E> {
    // holds the stack elements
    private E[] S;

    // index to top element
    private int top = -1;

    // constructor
    public ArrayStack(int capacity) {
        S = (E[]) new Object[capacity];
    }

    public E pop() throws EmptyStackException {
        if (isEmpty())
            throw new EmptyStackException("Empty stack: cannot pop");
        E temp = S[top];
        // facilitate garbage collection:
        S[top] = null;
        top = top - 1;
        return temp;
    }

    // other methods of Stack interface
}
```

© 2010 Goodrich, Tamassia
public class Tester {

    // … other methods

    public intReverse(Integer a[]) {
        Stack<Integer> s;
        s = new ArrayStack<Integer>();
        s = new ArrayStack<Integer>();

        … (code to reverse array a) …
    }

    public floatReverse(Float f[]) {
        Stack<Float> s;
        s = new ArrayStack<Float>();

        … (code to reverse array f) …
    }

© 2010 Goodrich, Tamassia
Parentheses Matching

- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["]
  - correct: ( )(( )){([ ( ]))}
  - correct: ((( )( )){([ ( ]))}
  - incorrect: )(( )){([ ( ]))}
  - incorrect: ({{ ]})
  - incorrect: (}
Parentheses Matching Algorithm

**Algorithm** ParenMatch(X,n):

**Input:** An array X of n tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number

**Output:** true if and only if all the grouping symbols in X match

Let S be an empty stack

for i=0 to n-1 do
  if X[i] is an opening grouping symbol then
    S.push(X[i])
  else if X[i] is a closing grouping symbol then
    if S.isEmpty() then
      return false \{nothing to match with\}
    if S.pop() does not match the type of X[i] then
      return false \{wrong type\}
    if S.isEmpty() then
      return true \{every symbol matched\}
  else return false \{some symbols were never matched\}
The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?
Tag Matching Algorithm (in Java)

```java
import java.io.*;
import java.util.Scanner;
import net.datastructures.*;

/** Simplified test of matching tags in an HTML document. */
public class HTML {

/** Strip the first and last characters off a <tag> string. */
public static String stripEnds(String t) {
    if (t.length() <= 2) return null;  // this is a degenerate tag
    return t.substring(1, t.length() - 1);
}

/** Test if a stripped tag string is empty or a true opening tag. */
public static boolean isOpeningTag(String tag) {
    return (tag.length() == 0) || (tag.charAt(0) != '/');
}

```

© 2010 Goodrich, Tamassia
Tag Matching Algorithm (cont.)

/** Test if stripped tag1 matches closing tag2 (first character is '/'). */
public static boolean areMatchingTags(String tag1, String tag2) {
    return tag1.equals(tag2.substring(1)); // test against name after '/'
}

/** Test if every opening tag has a matching closing tag. */
public static boolean isHTMLMatched(String[] tag) {
    Stack<String> S = new NodeStack<String>(); // Stack for matching tags
    for (int i = 0; (i < tag.length) && (tag[i] != null); i++) {
        if (isOpeningTag(tag[i]))
            S.push(tag[i]); // opening tag; push it on the stack
        else {
            if (S.isEmpty())
                return false; // nothing to match
            if (!areMatchingTags(S.pop(), tag[i]))
                return false; // wrong match
        }
    }
    if (S.isEmpty()) return true; // we matched everything
    return false; // we have some tags that never were matched
}
Tag Matching Algorithm (cont.)

public final static int CAPACITY = 1000; // Tag array size
/* Parse an HTML document into an array of html tags */
public static String[] parseHTML(Scanner s) {
    String[] tag = new String[CAPACITY]; // our tag array (initially all null)
    int count = 0; // tag counter
    String token; // token returned by the scanner s
    while (s.hasNextLine()) {
        while ((token = s.findInLine("<[^>]*>")) != null) // find the next tag
            tag[count++] = stripEnds(token); // strip the ends off this tag
        s.nextLine(); // go to the next line
    }
    return tag; // our array of (stripped) tags
}
public static void main(String[] args) throws IOException { // tester
    if (isHTMLMatched(parseHTML(new Scanner(System.in))))
        System.out.println("The input file is a matched HTML document.");
    else
        System.out.println("The input file is not a matched HTML document.");
}
Evaluating Arithmetic Expressions

\[14 - 3 \times 2 + 7 = (14 - (3 \times 2)) + 7\]

Operator precedence

* has precedence over +/-

Associativity

operators of the same precedence group evaluated from left to right

Example: \((x - y) + z\) rather than \(x - (y + z)\)

Idea: push each operator on the stack, but first pop and perform higher and equal precedence operations.
Algorithm for Evaluating Expressions

Two stacks:
- opStk holds operators
- valStk holds values
- Use $ as special “end of input” token with lowest precedence

Algorithm doOp()

\[ x \leftarrow \text{valStk.pop(); } \]
\[ y \leftarrow \text{valStk.pop(); } \]
\[ \text{op} \leftarrow \text{opStk.pop(); } \]
\[ \text{valStk.push}( y \text{ op } x ) \]

Algorithm repeatOps( refOp ):

\[ \text{while ( } \text{valStk.size()} > 1 \wedge \text{prec(refOp)} \leq \text{prec(opStk.top())) } \]
\[ \text{doOp() } \]

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)
Output: the value of the expression

\[ \text{while there’s another token } z \]
\[ \text{if isNumber(z) then } \]
\[ \text{valStk.push(z)} \]
\[ \text{else } \]
\[ \text{repeatOps(z); } \]
\[ \text{opStk.push(z)} \]
\[ \text{repeatOps($); } \]
\[ \text{return valStk.top()} \]
Algorithm on an Example Expression

14 \leq 4 - 3 \ast 2 + 7

Operator $\leq$ has lower precedence than $+/-$
Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an array $X$, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high

<table>
<thead>
<tr>
<th>X</th>
<th>6</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
Quadratic Algorithm

Algorithm $spans1(X, n)$

Input array $X$ of $n$ integers

Output array $S$ of spans of $X$

$S \leftarrow$ new array of $n$ integers

for $i \leftarrow 0$ to $n - 1$ do

$s \leftarrow 1$

while $s \leq i \land X[i - s] \leq X[i]$

$s \leftarrow s + 1$

$S[i] \leftarrow s$

return $S$

Algorithm $spans1$ runs in $O(n^2)$ time
We keep in a stack the indices of the elements visible when “looking back”

We scan the array from left to right

- Let $i$ be the current index
- We pop indices from the stack until we find index $j$ such that $X[i] < X[j]$
- We set $S[i] \leftarrow i - j$
- We push $x$ onto the stack
Linear Algorithm

Each index of the array
- Is pushed into the stack exactly one
- Is popped from the stack at most once

The statements in the while-loop are executed at most $n$ times

Algorithm $spans2(X, n)$

```
S ← new array of $n$ integers
A ← new empty stack
for $i$ ← 0 to $n - 1$ do
    while (¬A.isEmpty() ∧
        $X[A.top()] \leq X[i]$ ) do
        A.pop()
    if A.isEmpty() then
        $S[i] ← i + 1$
    else
        $S[i] ← i - A.top()$
A.push($i$
return $S$
```

Algorithm $spans2(X, n)$ runs in $O(n)$ time

© 2010 Goodrich, Tamassia