1 Since \(-2\) is a root, \(x + 2\) is a factor of the polynomial. So the first step is to divide \(x^3 - 2x^2 - 9x - 2\) by \(x + 2\). There are many ways to do this, but the most efficient is called “synthetic division” (see Wikipedia for a detailed explanation). The division process is as follows:

\[
\begin{array}{c|cccc}
-2 & 1 & -2 & -9 & -2 \\
-2 & 0 & 8 & 2 \\
\hline
1 & -4 & -1 & 0 \\
\end{array}
\]

which means that the quotient is \(x^2 - 4x - 1\) (with a remainder of 0). The additional roots of the original polynomial are the roots of this quotient, which, by the quadratic formula, are \(2 + \sqrt{5}\) and \(2 - \sqrt{5}\).

2 Since the cosine function has a period of \(2\pi\), \(\cos(x + 2\pi) = \cos x\). Shifting the argument of cosine by half the period changes the sign, so \(\cos(x - \pi) = -\cos x\). Since sine is an odd function, \(\sin(x - \pi/2) = -\sin(\pi/2 - x)\); and this is \(-\cos(x)\) because the sine and cosine of complementary angles are equal. A useful identity to know is \(\cos^2 x = (1 + \cos 2x)/2\); thus \(2\cos^2 x = 1 + \cos 2x\) and \(2\cos^2 x - \cos 2x = 1\). Finally, \(\sin x \tan x = \sin^2 x / \cos x = (1 - \cos^2 x) / \cos x = \sec x - \cos x\). Combining all these reduces the original expression to \(1 + \sec x\).

3 If you’ve studied linear algebra, you know a systematic method for solving systems of linear equations like these. Otherwise, a straightforward approach is to eliminate one variable at a time. In this case, \(x\) is already missing in the last two equations. We can eliminate \(y\) by subtracting the second equation from the third, yielding \(2z = 3\); thus \(z = 3/2\). Plugging this into either the second or third equation results in \(y = 1\), and putting both values in the first equation gives \(x = 1/2\).

4 Since we can’t easily solve for \(y\), this calls for implicit differentiation. Applying \(d/dx\) to the given equation results in

\[
3y^2 \frac{dy}{dx} + 3 \frac{dy}{dx} - 3x^2 = 0.
\]

Dividing by 3 and factoring \(dy/dx\) from the first two terms gives

\[
(y^2 + 1) \frac{dy}{dx} - x^2 = 0,
\]

so \(\frac{dy}{dx} = \frac{x^2}{y^2 + 1}\).
This problem calls for integration by parts. This technique is taught in a number of different ways; here we use the variables \( u \) and \( v \) and the formula

\[
\int u dv = uv - \int v du.
\]

In this problem, we apply integration by parts twice; first with \( u = x^2 \) and \( v = e^x \):

\[
\int x^2 e^x \, dx = x^2 e^x - 2 \int x e^x \, dx.
\]

For the second integral, we take \( u = x \) and \( v = e^x \), so the complete result is

\[
x^2 e^x - 2(xe^x - \int e^x \, dx) = x^2 e^x - 2xe^x + 2e^x
\]

\[
= (x^2 - 2x + 2)e^x.
\]

Ignoring the limits of integration for the moment, we use integration by parts with \( u = e^x \) and \( v = \sin x \):

\[
\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx.
\]

Using integration by parts on the second integral (with \( u = e^x \), \( v = -\cos x \)),

\[
\int e^x \sin x \, dx = -e^x \cos x + \int e^x \cos x \, dx,
\]

which leaves us with the integral we started with. But this is progress, because the sign has changed; substituting the second equation into the first results in

\[
\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx,
\]

so adding the integral and dividing by 2 gives

\[
\int e^x \cos x \, dx = (e^x \sin x + e^x \cos x) / 2.
\]

Evaluating the right-hand side between the values \( x = 0 \) and \( x = \pi/3 \) gives the final answer:

\[
\int_0^{\pi/3} e^x \cos x \, dx = \frac{\sqrt{3} + 1}{4} e^{\pi/3} - \frac{1}{2}.
\]

In order to use partial fractions, we need to factor the denominator. Now \( x^3 - 3x^2 + 2x = x(x^2 - 3x + 2) \), and the quadratic factor is just \((x - 1)(x - 2)\). Thus we need to find constants \( A, B \) and \( C \) so that

\[
\frac{2x + 1}{x(x - 1)(x - 2)} = \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x - 2}.
\]
Multiplying through by \(x(x - 1)(x - 2)\) gives

\[
2x + 1 = A(x - 1)(x - 2) + Bx(x - 2) + Cx(x - 1)
= A(x^2 - 3x + 2) + B(x^2 - 2x) + C(x^2 - x)
= (A + B + C)x^2 + (-3A - 2B - C)x + 2A.
\]

Equating the coefficients on the two sides, we have

\[
A + B + C = 0
-3A - 2B - C = 2
2A = 1.
\]

From the last equation, \(A = 1/2\), so we are left with

\[
B + C = -1/2
-2B - C = 7/2
\]

Adding these gives \(-B = 3\), so \(B = -3\) and thus \(C = 5/2\). The final result is then

\[
\frac{2x + 1}{x(x - 1)(x - 2)} = \frac{1/2}{x} - \frac{3}{x - 1} + \frac{5/2}{x - 2}.
\]