An equitable $k$-coloring of a graph is a proper $k$-coloring in which no two color classes differ in size by more than one. In 1970, Hajnal and Szemerédi proved that every graph with maximum degree less than $k$ has an equitable $k$-coloring. This theorem is easily seen to be best possible: complete graphs, odd cycles and balanced complete bipartite graphs with odd sized parts are all not equitably $k$-colorable when $k$ is the maximum degree. In 1994, Chen, Lih and Wu conjectured that these are the only connected graphs that are not equitably $k$-colorable and have maximum degree at most $k$. This conjecture is still open, but a few special cases have been proved.

In this talk, we will discuss an Ore-type result which implies the Chen-Lih-Wu conjecture for $k$-equitable colorings in graphs with $3k$ vertices as well as other results and conjectures related to the Hajnal-Szemerédi Theorem.

This is joint work with Alexandr Kostochka, H.A. Kierstead and Elyse Yeager.