On Three Hamiltonian Problems
For Triple Systems

Andrzej Ruciński  
A. Mickiewicz University, Poznań, Poland
and
Emory University, Atlanta

Abstract

The extremal theory of hypergraphs has become a fashionable topic nowadays. Following this trend I will discuss three problems (with only partial solutions) on the existence of Hamiltonian cycles in \( k \)-uniform hypergraphs, or \( k \)-graphs for short, with emphasis on triple systems \( (k = 3) \).

The first problem asks for the minimum number of edges in a maximally non-Hamiltonian \( k \)-graph – the so called saturation number. The second, Dirac-type problem, is to determine the minimum vertex degree guaranteeing a Hamiltonian cycle in a \( k \)-graph. Finally, taking an anti-Ramsey approach, we study the restrictions under which an edge-coloring of the complete \( k \)-graph \( K_n^{(k)} \) yields a rainbow Hamiltonian cycle.

During my talk I will present recent results from joint projects (some still in progress) with Andrzej Żak, with Endre Szemerédi, Mathias Schacht, and Vojtech Rödl, and with Alan Frieze and Andrzej Dudek.