1. (2 points) In the table below we list the probabilities that a certain kind of dog will have precisely \( n \) offsprings in one litter.

<table>
<thead>
<tr>
<th>the number of offsprings ( n )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>the probability ( f(n) )</td>
<td>0.26</td>
<td>0.20</td>
<td>0.32</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Find the mean and the standard deviation for the number of offsprings in one litter.

**Answer:** \( \mu = 1.5, \quad \sigma = 1.1. \)

2. (3 points) Among a person’s ten pairs of socks, four pairs need mending. If he randomly picks three pairs of sock to take along on a trip, what are the probabilities that:

(a) one pair of the socks will need mending?

**Answer:** Since we deal with a hypergeometric distribution we have

\[
f(1) = \frac{\binom{4}{1} \binom{6}{2}}{\binom{10}{3}} = 0.5.
\]

(b) at least one pair will need mending?

**Answer:**

\[
1 - f(0) = 1 - \frac{\binom{4}{0} \binom{6}{3}}{\binom{10}{3}} = \frac{5}{6} \sim 0.833.
\]
(c) fewer than three pairs will need mending?

Answer:

\[ 1 - f(3) = 1 - \frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}} = \frac{29}{30} \sim 0.967. \]

3. (2 points) A psychologist wants to choose 5 persons for a team on which she will conduct some psychological experiment. Suppose that these five persons are selected at random from 478 students of a certain college, of whom 156 are men and 322 are women.

(a) What is the probability that the team will consist of two men and three women?

Answer:

\[ \frac{\binom{156}{2}\binom{322}{3}}{\binom{478}{2}}. \]

(b) Approximate the above probability (that the team will consist of two men and three women) using the binomial approximation.

Answer: We approximate the above hypergeometric distribution (with parameters \( n = 5, a = 156, b = 322 \)) by the binomial distribution with \( n = 5, p = \frac{a}{a+b} = \frac{156}{478} \sim 0.326. \) Thus, the probability we are looking for are given by

\[ f(2) = \binom{5}{2}(0.326)^2(1 - 0.326)^{5-2} \sim 0.326. \]
4. (4 points) If the number of blossoms on a rare cactus is a random variable having the Poisson distribution with $\lambda = 1.6$, what are the probabilities that such a cactus will have:

(a) no blossoms?

**Answer:** $f(0) = e^{-1.6} \sim 0.202$.

(b) at least two blossoms?

**Answer:** $1 - f(0) - f(1) \sim 1 - 0.202 - 0.323 = 0.475$.

(c) more than one but less than four blossoms?

**Answer:** $f(2) + f(3) \sim 0.258 + 0.138 = 0.396$.

5. (4 points) The number of customers to whom a restaurant serves breakfast on a weekday morning is a random variable with $\mu = 142$ and $\sigma = 12$.

(a) According to Chebyshev’s theorem, with what probability can we assert that between 112 and 172 customers will have breakfast there on a weekday morning?

**Answer:** The probability is at least $1 - \frac{1}{(2.5)^2} = 0.84$.

(b) Assume that the number of customers can be well approximated by the normal distribution with $\mu = 142$ and $\sigma = 12$. Use this fact to estimate the probability that the restaurant will serve a breakfast on a weekday morning to at least 126 and at most 145 customers.

**Answer:** You have to use the continuity correction here. Since $\frac{145.5 - 142}{12} = 1.38$ and $\frac{125.5 - 142}{12} = -1.38$, the probability we are looking for is equal to $0.4162 + 0.1141 = 0.5303$. 
(c) Similarly as in (b), use the normal approximation to estimate the probability that the restaurant will serve a breakfast on a weekday morning to at most 138 customers.

**Answer:** Again, we should use the continuity correction. Then we have \( \frac{138.5 - 142}{12} = -0.29 \) and for the probability we get \( 0.5 - 0.1141 = 0.3859 \).

6. (2 points) If the probability is 0.22 that a set of tennis will go into a tie breaker, what is the probability that at most two of five sets will go into tie breakers?

**Answer:** We deal with the binomial distribution so
\[
\begin{align*}
f(0) &= \binom{5}{0}(0.22)^0(1 - 0.22)^5 \sim 0.2887, \\
f(1) &= \binom{5}{1}(0.22)^1(1 - 0.22)^4 \sim 0.4072, \\
f(2) &= \binom{5}{2}(0.22)^2(1 - 0.22)^3 \sim 0.2297,
\end{align*}
\]
and for the probability that at most two tie breakers occur we get \( f(0) + f(1) + f(2) \sim 0.9256 \).

7. (3 points) Assume that the number of miles a driver gets on a set of radial tires is normally distributed with a mean of 30,000 miles and a standard deviation of 5000 miles. What is the probability that a driver using such a set of tires will get:

(a) at least 33,000 miles?

**Answer:** You should not use the continuity correction here! Thus, since \( \frac{33000 - 30000}{5000} = 0.6 \), the answer is \( 0.5000 - 0.2257 = 0.2743 \).

(b) less than 29,000 miles?

**Answer:** Again, no continuity correction is needed. Hence, since \( \frac{29000 - 30000}{5000} = -0.2 \), for the probability we get \( 0.5000 - 0.0793 = 0.4207 \).