Math 107. Review for the first midterm  
Solutions to the problems which are not solved in the book.

R.2 (a) At least \(1 - \frac{1}{(1.2)^2} = 0.3056\).  
(b) At least \(1 - \frac{1}{(6.25)^2} = 0.9744\).  
(c) At least \(1 - \frac{1}{17^2} = 0.9917\).  

R.13 (a) \(\bar{x} = \frac{0.51}{8} = 0.06375\), \(s = \sqrt{0.000021907} \sim 0.0044\).  
(b) The median is 0.0625, the range is 0.012.  

R.19 By Chebyshev’s inequality, the probability is at least \(1 - \frac{1}{4^2} = \frac{15}{16}\).  

R.54 \(\frac{4}{10} \cdot \frac{3}{5} \cdot \frac{2}{8} = \frac{1}{30}\).  
R.55 (a) \(4 \cdot 3 \cdot 4 = 48\).  
(b) \(3 \cdot 4 = 12\).  
(c) \(4 \cdot 3 \cdot 2 = 24\).  

R.58 \(0.20 + 0.25 - 0.15 = 0.30\).  

R.59 \(7 \cdot 6 \cdot 5 = 210\) ways.  

R.62 (a) \(P(A) = \frac{180}{380} = \frac{9}{31} \sim 0.474\).  
(b) \(P(R') = \frac{220}{380} = \frac{11}{19} \sim 0.586\).  
(c) \(P(A \cap R) = \frac{20}{380} = \frac{1}{19} \sim 0.053\).  
(d) \(P(A|R) = \frac{20/380}{180/380} = \frac{2}{3} \sim 0.286\).  
(e) \(P(A' \cup R) = \frac{220/380}{180/380} = \frac{11}{19} \sim 0.579\).  
(f) \(P(R'|A) = \frac{160/380}{180/380} = \frac{8}{9} \sim 0.889\).  

R.69 (a) \(\frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{256} \sim 0.0039\).  
(b) \(\frac{13}{52} \cdot \frac{12}{52} \cdot \frac{11}{52} \cdot \frac{10}{49} = \frac{17160}{6997400} \sim 0.0026\).  

R.71 (a) \(\binom{6}{4} = 15\) ways.  
(b) \(\binom{6}{2} \cdot \binom{6}{2} = 225\) ways.  
(c) \(\binom{12}{4} = 495\) ways.  

Problem.  
We select at random 5 cards from a deck of 52 cards. Find the probability that:  
(i) we will have at most one ace;  

Answer: \(\frac{\binom{48}{5} + \binom{4}{1} \binom{48}{4}}{\binom{52}{5}}\).  

(ii) we will have precisely one spade and two clubs;  

Answer: \(\frac{\binom{13}{1} \binom{13}{2} \binom{26}{2}}{\binom{52}{5}}\).  

(iii) we will have three diamonds and suits of the two remaining cards are different.  

Answer: \(\frac{\binom{13}{3} \binom{3}{2} \binom{13}{1} \binom{13}{1}}{\binom{52}{5}}\).