1. [Erdős–Szekeres] Every sequence $a_0, \ldots, a_{n^2}$ of integers contains a monotonic subsequence with $n + 1$ elements, that is, there are $0 \leq i_0 < \cdots < i_n \leq n^2$ with $a_{i_0} \leq \cdots \leq a_{i_n}$ or with $a_{i_0} > \cdots > a_{i_n}$.

   (i) Deduce the above result from Dilworth’s theorem.

   (ii) Prove the above result from first principles. [Hint. For each $i$, consider the pair $(u_i, d_i)$, where $u_i$ is the number of elements in an increasing subsequence starting with element $a_i$, and $d_i$ is the number of elements in a decreasing subsequence starting at $a_i$.]

2. (i) Prove that any $n + 1$ elements of $[2n] = \{1, \ldots, 2n\}$ contains two elements $x, y$ that are relatively prime.

   (ii) Prove that any $n + 1$ elements of $[2n]$ contains two elements $x, y$ with $x \mid y$ (x divides y). You can do this by considering the factorization $2^k m$ ($m$ odd) for the integers in question. Describe the poset underlying this problem, determine a minimum chain decomposition and a maximum antichain.