1. Prove that there is a colouring of the natural numbers with red and blue such that no infinite arithmetic progression is monochromatic. \[ \text{[Hint. Observe that there are only countably many arithmetic progression; consider an enumeration of them.]} \]

2. Prove that the infinite subsets of $\mathbb{N}$ may be coloured with red and blue in such a way that no infinite subset $S \subseteq \mathbb{N}$ has all its infinite subsets coloured with the same colour. \[ \text{[Hint. Write } A \sim B \text{ if the symmetric difference } A \Delta B \text{ is finite; colour the sets so that, if } A = B \setminus \{b\}, \text{ then } A \text{ and } B \text{ have different colours.]} \]

3. Prove that there is a colouring of the pairs (2-element subsets) of $\mathbb{R}$ with red and blue such that no uncountable subset $S \subseteq \mathbb{R}$ has all its pairs of the same colour. \[ \text{[Hint. Consider a well ordering } \preceq \text{ of the reals. Colour a pair } \{x, y\} \text{ blue if } x \preceq y \text{ and } x \leq y \text{ (} \leq \text{ is the usual order on the reals).]} \]

4. (i) Suppose a family of sets $\mathcal{F} \subseteq \mathcal{P}(\mathbb{N}) = 2^\mathbb{N}$ is such that the symmetric difference $F \Delta F'$ is finite for all distinct $F, F' \in \mathcal{F}$. Prove that $\mathcal{F}$ is countable.

(ii) A family of sets $\mathcal{F}$ is called quasi-disjoint if the intersection $F \cap F'$ is finite for all distinct $F, F' \in \mathcal{F}$. Prove that there is an uncountable quasi-disjoint family of sets $\mathcal{F} \subseteq \mathcal{P}(\mathbb{N}) = 2^\mathbb{N}$. 

\[ Date: \text{ April 27, 2003.} \]
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